Assume two populations with unknown means, μ_1 and μ_2, but known variances σ_1^2 and σ_2^2. Sample 100 observations from each population. From the data:

$$n_1 = n_2 = 100$$

$$\sigma_1^2 = 1400, \quad \sigma_2^2 = 2200$$

$$\bar{X}_1 = 780 \qquad \bar{X}_2 = 810$$

95% confidence interval to Estimate $\mu_1 - \mu_2$

$$\bar{X}_1 - \bar{X}_2 \pm z_{\frac{.05}{2}} \cdot \sigma_{\bar{X}_1 - \bar{X}_2}$$

$$\bar{X}_1 - \bar{X}_2 \pm z_{.025} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$780 - 810 \pm 1.96 \sqrt{\frac{1400}{100} + \frac{2200}{100}}$$

$$-30 \pm 1.96\,(6)$$

$$-30 \pm 11.76$$

$$(-41.76,\ -18.24)$$

Do the data indicate that the means are different?

$$\left.\begin{array}{l} H_0: \mu_1 - \mu_2 = 0 \\ H_A: \mu_1 - \mu_2 \neq 0 \end{array}\right\} \text{same} \quad \begin{array}{l} H_0: \mu_1 = \mu_2 \\ H_A: \mu_1 \neq \mu_2 \end{array}$$

①

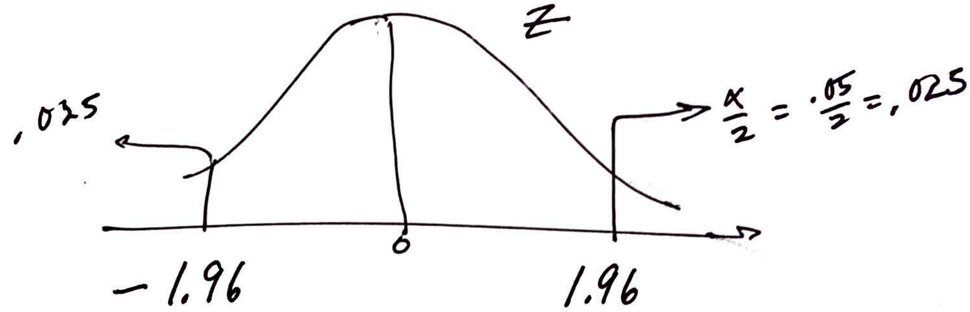
2. $$z_{calc} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sigma_{\bar{X}_1 - \bar{X}_2}} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} =$$

$$= \frac{780 - 810 - 0}{\sqrt{\frac{1400}{100} + \frac{2200}{100}}} = \frac{-30}{6} = -5$$

3. If H_0 is true, $\mu_1 = \mu_2$, then $z_{calc} \sim Z$

4. Decision.

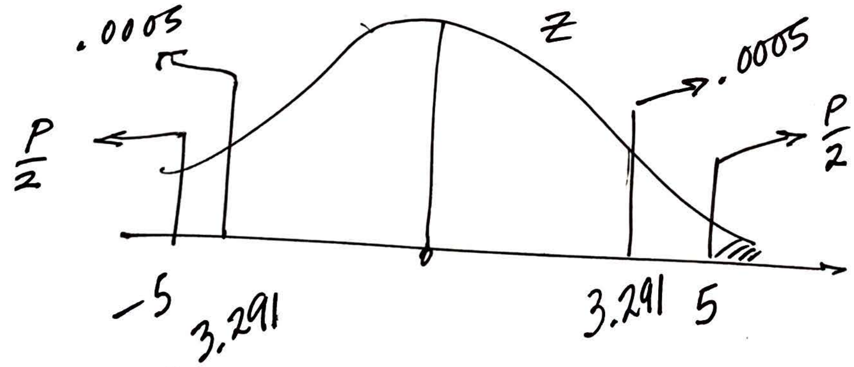
Rejection Region. $\alpha = .05$



Reject Ho at $\alpha = .05$ if $z_{calc} < -1.96$ or $z_{calc} > 1.96$.

Since $z_{calc} = -5 < -1.96 \Rightarrow$ Reject Ho at $\alpha = .05$

P-value $= P(Z < -5) + P(Z > 5)$



$\frac{P}{2} < .0005 \Rightarrow P < .001$

Strongly reject Ho since $p < .001$.
Conclude, yes, the data do indicate that the means are different.

Do the data indicate that μ_2 exceeds μ_1 by more than 25 units?

$H_0: \mu_2 - \mu_1 = 25$

$H_A: \mu_2 - \mu_1 > 25$

$H_0: \mu_1 - \mu_2 = -25$

① $H_A: \mu_1 - \mu_2 < -25$

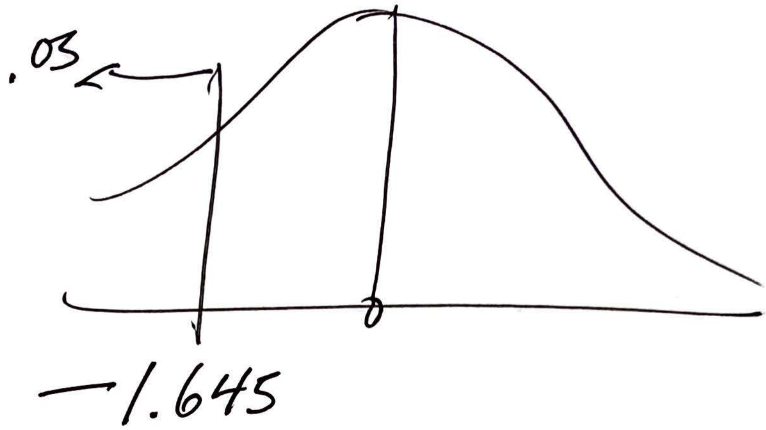
② $$z_{calc} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sigma_{\bar{X}_1 - \bar{X}_2}} = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{780 - 810 - (-25)}{\sqrt{\frac{1400}{100} + \frac{2200}{100}}} = \frac{-30 + 25}{6} = \frac{-5}{6} = -.8\overline{3}$$

3. If H_0 is true, $\mu_1 - \mu_2 = -25$ then

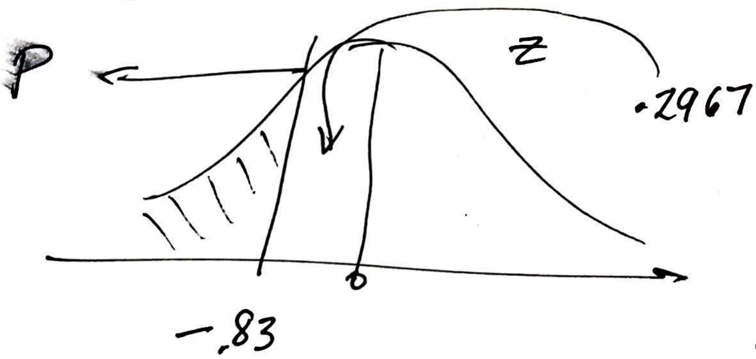
$z_{calc} \sim Z$ or $N(0,1)$.

4. Decision. $\alpha = .05$.



$z_{calc} = -.8\bar{3} \nless -1.645 \Rightarrow$ Do not reject H_0 at $\alpha = .05$.

P-value, $P = P(Z < -.8\bar{3})$



$P = .5 - .2967 = .2033$

	.03
.8	.2967

With $p = .2033 \Rightarrow$ Do not Reject H_0.
Conclude, the data do not indicate that μ_2 exceeds μ_1 by more than 25 units.