STATISTICS 2023 **NAME IN INK/PRINT** Key

EXAM TWO **SIGNATURE IN INK** _____

SPRING 2019 **CWID IN INK** _____

TRUE OR FALSE. Answer with a capital T or F. **(4 points each)**

T 1. A continuous random variable has probability on intervals of values, but no probability on specific values.

F 2. The mass function for a Poisson random variable can be symmetric, left skewed, or right skewed, depending on the value of the parameter for the variable.

T 3. A continuous random variable has probability that described with a probability density function and the area under the curve indicates the probability for a specific interval of values.

F 4. A variable that has a normal distribution has approximately 68% of its probability on values that are outside an interval that is within one standard deviation of the mean.

T 5. The mean of the sample mean is equal to the mean of the sampled population and the standard deviation of the sample mean is equal to the standard deviation of the sampled population divided by square-root of n.

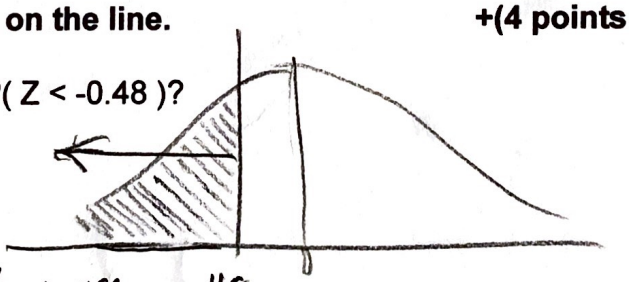
Z-TABLE QUESTIONS. Write the answer on the line. **+(4 points each)**

0.31561 6. What is the value of P(Z < -0.48)?

$P(z < -.48) =$

$= P(z > .48) =$

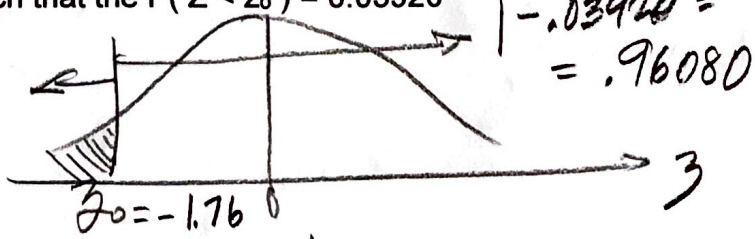
$= 1 - P(z < .48) = 1 - .68439 = .31561$



-1.76 7. What is the value of z_o, such that the P(Z < z_o) = 0.03920

.03920

$1 - .03920 = .96080$

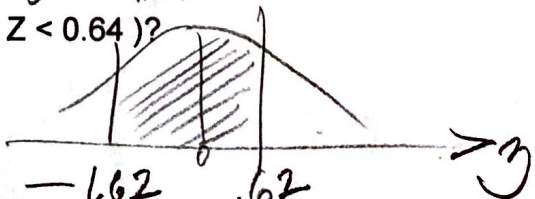


0.68629 8. What is the value of P(-1.62 < Z < 0.64)?

$P(z < .64) - P(z < -1.62)$

$= P(z < .64) - P(z > 1.62)$

$= P(z < .64) - [1 - P(z < 1.62)] = .73891 - [1 - .94738] = 0.68629$



STATE THE ANSWER. Write the answer on the line. (4 points each)

$35,000. 9. A promoter is planning an outdoor concert. If it rains the promoter will lose $22,000, but if it does not rain the profit will be $54,000. If the chance of rain is .25, what is the expected profit from this concert?

X	-22,000	54,000
P(X)	.25	.75

$E[X] = M = \sum x p(x) = -22,000(.25) + 54,000(.75)$
$= 35,000$

0.8503 10. A new type of car designed by an American car company has a 90% chance of exceeding the MPG fuel use rating on the window sticker. Out of seven cars of this type purchased by local taxi service, what is the probability that more than 5 of them will exceed the MPG rating on the window sticker? Round your answer to 4 digits past the decimal.

$X \sim Bi(n=7, p=.90)$

$P(x>5) = P(x \geq 6) = P(x=6) + P(x=7) = \binom{7}{6}.9^6(1-.9)^1 + \binom{7}{7}.9^7(1-.9)^0 =$
$= 0.3720087 + 0.4782969 = .8503.$

0.1756 11. An accounting firm is investigating 20 corporations. The probability that the firm will identify some accounting problem in any one of these corporations is 0.15. What is the probability that the firm will identify accounting problems in at most 1 of these corporations? Round your answer to 4 digits past the decimal. $X \sim Bi(n=20, p=0.15)$

$P(X \leq 1) = P(X=0) + P(X=1)$
$= \binom{20}{0}.15^0(1-.15)^{20} + \binom{20}{1}.15^1(1-.15)^{19} = 0.038759531 + 0.136718345$

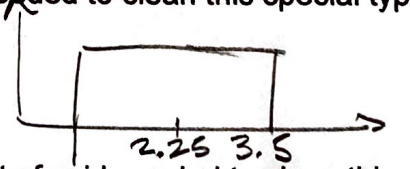
0.9666 12. If there are 3.4 vehicle accidents on a major highway each month, what is the probability that there will be one or more accidents in a randomly chosen month? Round your answer to 4 digits past the decimal. $X \sim Poi(\lambda = 3.4)$

$P(X \geq 1) = 1 - P(X=0) = 1 - \frac{3.4^0 e^{-3.4}}{0!} = 0.96662673$

Assume that amount of acid needed to clean a specific type of an automotive cylinder is a uniformly distributed random variable between 1 and 3.5 liters. Use this information to answer the next three questions. X = amount of acid needed, $X \sim Unif\ Cont(1, 3.5)$

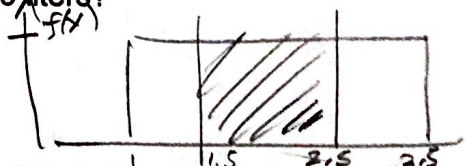
2.25 13. What is the average amount of acid needed to clean this special type of an automotive cylinder?

$M = \frac{c+d}{2} = \frac{1+3.5}{2} = 2.25$



0.4 14. What is the probability that the amount of acid needed to clean this special type of an automotive cylinder is between 1.5 and 2.5 liters?

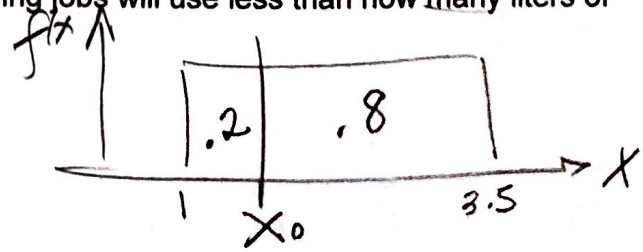
$P(1.5 < X < 2.5) = Bh = (2.5 - 1.5)\frac{1}{2.5} = 1\left(\frac{1}{2.5}\right) = 0.4$



1.5 15. Twenty percent of the cleaning jobs will use less than how many liters of acid?

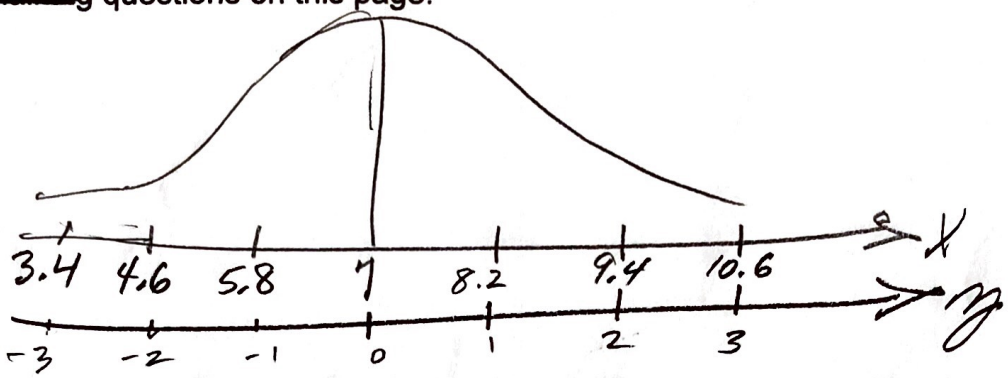
$X_0 = 1 + .2(2.5) = 1.5$

or $X_0 = 3.5 - .8(2.5) = 1.5$



STATE THE ANSWER. Write the answer on the line. **(4 points each)**

At a large hospital complex the time required for an essential prescription medicine to be delivered from the pharmacy to a patient's hospital room is a normally distributed random variable with a mean of 7 minutes and a standard deviation of 1.2 minutes. Use this information to address the ~~remaining~~ questions on this page.



0.88045

16. What is the probability that the time required to deliver the prescription medicine is between 4.36 and 8.5 minutes?

$P(4.36 < X < 8.5) =$

$= P\left(\frac{4.36-7}{1.2} < \frac{X-M}{\sigma} < \frac{8.5-7}{1.2}\right) =$

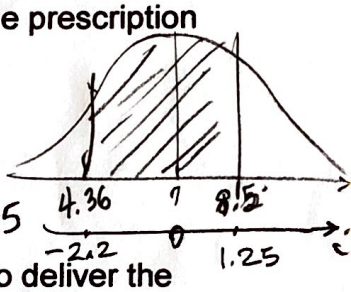
$= P(-2.2 < Z < 1.25) =$

$\Rightarrow P(Z<1.25) - P(Z<-2.2) =$

$= P(Z<1.25) - P(Z>2.2) =$

$= P(Z<1.25) - [1 - P(Z<2.2)]$

$= 0.89435 - [1 - .98610] = 0.88045$



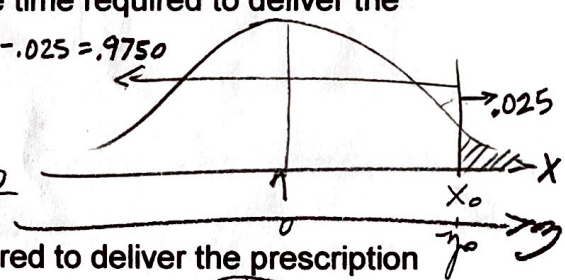
9.352

17. Two and one-half percent of the time, the time required to deliver the prescription medicine is more than how many minutes?

Find X_0, such that $P(X > X_0) = .025$

Soln. 1. Find z_0, $P(Z > z_0) = 0.025 \Rightarrow z_0 = 1.96$

2. Calc $X_0 = M + z_0\sigma = 7 + 1.96(1.2) = 9.352$



0.12747

18. What is the probability that the time required to deliver the prescription medicine is between 8.32 and 9.88 minutes?

$P(8.32 < X < 9.88) =$

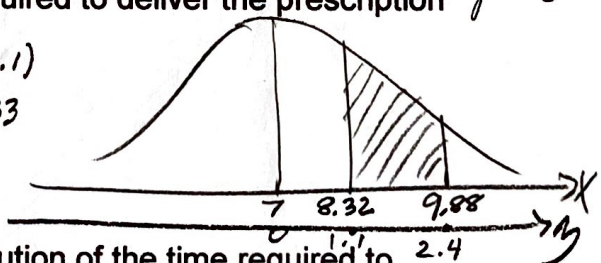
$= P\left(\frac{8.32-7}{1.2} < \frac{X-M}{\sigma} < \frac{9.88-7}{1.2}\right) =$

$= P(1.1 < Z < 2.4) =$

$\Rightarrow P(Z<2.4) - P(Z<1.1)$

$= 0.99180 - 0.86433$

$= 0.12747$



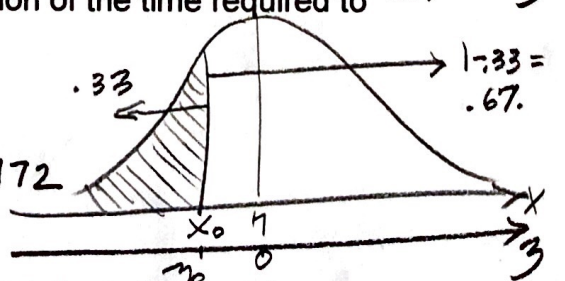
6.472

19. The thirty-third percentile for this distribution of the time required to deliver the prescription medicine is equal to what value?

Find X_0, such that $P(X < X_0) = .33$

Soln. 1. Find z_0, $P(Z < z_0) = .33 \Rightarrow z_0 = -.44$

2. Calc $X_0 = M + z_0\sigma = 7 + (-.44)1.2 = 6.472$



0.00135

20. The patient gets a price reduction for slow delivery that requires more than 10.6 minutes. What is the probability that a patient will receive this price reduction?

$P(X > 10.6) =$

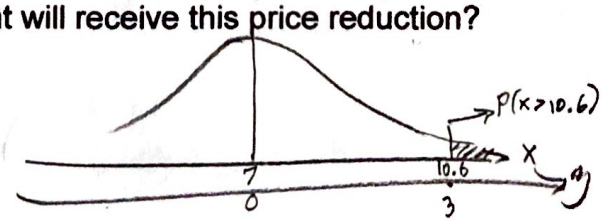
$= P\left(\frac{X-M}{\sigma} > \frac{10.6-7}{1.2}\right) =$

$= P(Z > 3) =$

$= 1 - P(Z < 3) =$

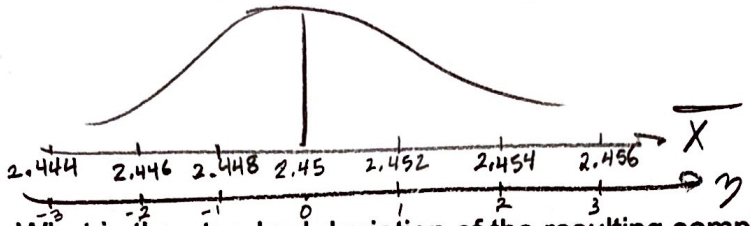
$= 1 - .99865 =$

$= 0.00135$



STATE THE ANSWER. Write the answer on the line. **(4 points each)**

A field medical device injects patients with a specific amount of fluid, but the process has a slight amount of variation. One setting on the device has a mean injection amount of 2.45ml with a standard deviation of 0.02ml. Assume that samples of 100 observations were repeatedly recorded from this setting on the field medical injection device. Consider the set of all sample means that would result from this repeated sampling process. Use this information to answer all the problems on this page. $\mu = 2.45$, $\sigma = 0.02$, $n = 100$, $\bar{X} \sim N(2.45, \sigma_{\bar{x}}^2)$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{.02}{\sqrt{100}} = .002$



0.002 21. What is the standard deviation of the resulting sample means from this setting on the field medical injection device?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{100}} = 0.002$$

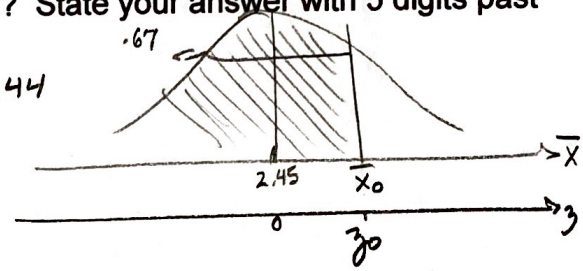
2.45088 22. Sixty-seven percent of the resulting sample means from this setting on the field medical injection device is less than how many ml? State your answer with 5 digits past the decimal. Find $\bar{x}_0$, such that $P(\bar{x} < \bar{x}_0) = .67$

Soln. 1. Find z_0, $P(z < z_0) = .67 \Rightarrow z_0 = 0.44$

2. Calc $\bar{x}_0 = \mu_{\bar{x}} + z_0 \sigma_{\bar{x}}$

$= 2.45 + (.44).002$

$= 2.45088$



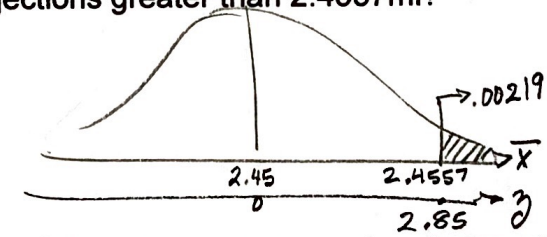
0.00219 23. The medicine being injected by the device can be toxic if too much is injected. What percent of the time is the mean of 100 injections greater than 2.4557ml?

$P(\bar{x} > 2.4557) =$

$= P\left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} > \frac{2.4557 - 2.45}{0.002}\right) =$

$= P(z > 2.85) =$

$\doteq 1 - P(z < 2.85) = 1 - .99781 = .00219$



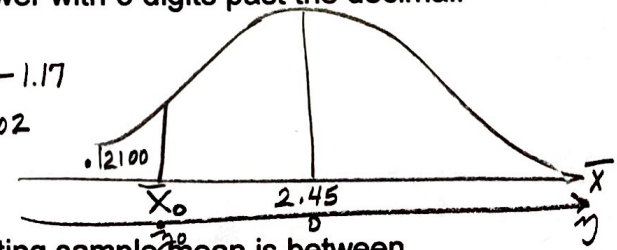
2.44766 24. The mean of 100 observations from this field medical injection device will be less than what value 12.1% of the time? State your answer with 5 digits past the decimal.

Find $\bar{x}_0$, such that $P(\bar{x} < \bar{x}_0) = .12100$

Soln. 1. Find z_0, $P(z < z_0) = .12100 \Rightarrow z_0 = -1.17$

2. Calc $\bar{x}_0 = \mu + z_0 \sigma_{\bar{x}} = 2.45 + (-1.17).002$

$= 2.44766$



0.81859 25. What is the probability that the resulting sample mean is between 2.448ml and 2.454ml?

$P(2.448 < \bar{x} < 2.454) =$

$= P\left(\frac{2.448 - 2.45}{.002} < \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} < \frac{2.454 - 2.45}{.002}\right) =$

$= P(-1 < z < 2) =$

$= P(z < 2) - P(z < -1) =$

$= P(z < 2) - [1 - P(z < 1)] =$

$= .97725 - [1 - .84134] =$

$= 0.81859$