

STATISTICS 2023

NAME IN PRINT

Key

EXAM TWO

SIGNATURE IN INK

FALL 2018

CWID IN INK

TRUE OR FALSE. Answer with a capital T or F.

(4 points each)

F 1. A discrete random variable has probability on intervals of values, but has no probability on specific values.

F 2. A Binomial random variable is right skewed if the probability of success on one trial is .9.

T 3. For a Poisson random variable, the mean and variance are always equal regardless of the value of the parameter for the variable.

T 4. A variable that has a normal distribution has half of its probability on values that are less than the mean and half of its probability on values that are more than the mean.

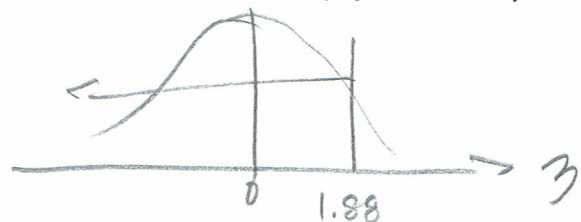
F 5. The mean of the sample mean is equal to the mean of the sampled population, but the variance of the set of all possible sample means is the original variance multiplied by n , the number of observations in the sample.

Z-TABLE QUESTIONS. Write the answer on the line.

(4 points each)

0.96995 6. What is the value of $P(Z < 1.88)$?

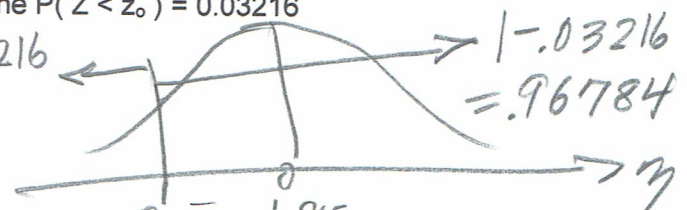
$$P(Z < 1.88) = 0.96995$$



-1.85

7. What is the value of z_0 , such that the $P(Z < z_0) = 0.03216$

0.03216

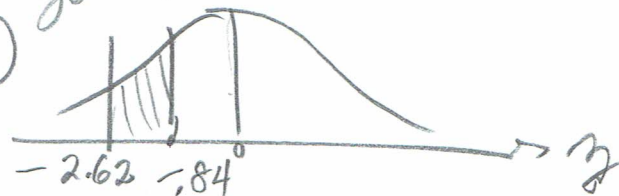


0.19605

8. What is the value of $P(-2.62 < Z < -0.84)$?

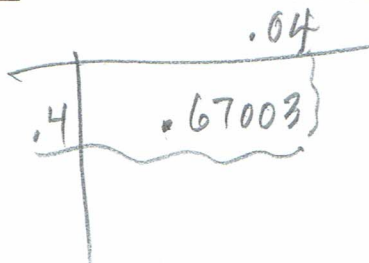
$$= P(Z < -2.62) - P(Z < -0.84)$$

$$= .99560 - .79955 = .19605$$

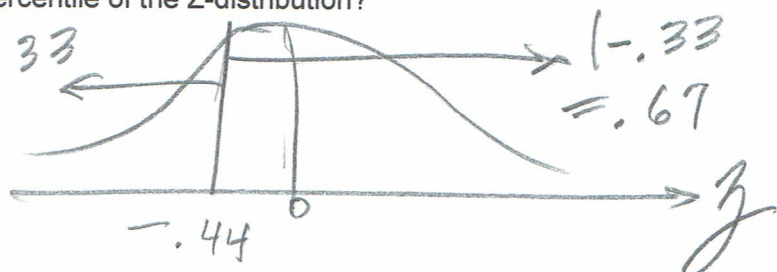


.44

9. What is the value of the 33rd percentile of the Z-distribution?



.33



STATE THE ANSWER. Write the answer on the line.

(4 points each)

3 10. The possible values of a discrete random variable are $x=2, 3$, and 6 , where the probability on each value is $1/x$, so the probability of 2 is $1/2$. What is the expected value of such a variable?

0.4

x	2	3	6
$P(x)$	$1/2$	$1/3$	$1/6$

$$M = 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{3}\right) + 6\left(\frac{1}{6}\right) = 3$$

11. Assume a uniform discrete random variable has 10 possible values. What would be the cumulative probability on four of the possible 10 values?

$$P(\text{each value}) = .10 \text{ so } P(4 \text{ events}) = .4$$

0.0598

12. A new type of car designed by a Japanese car company has a 12% chance of being recalled during the first 3 years of service. Your company plans to purchase a fleet of 8 of these cars, what is the probability that 3 or 4 of them will be recalled during the first 3 years of service? Round your answer to four digits past the decimal.

$$X \sim \text{Bi}(n=8, p=.12)$$

$$P(X=3) + P(X=4) = \binom{8}{3} \cdot .12^3 \cdot .88^5 + \binom{8}{4} \cdot .12^4 \cdot .88^4 =$$

0.5663

13. Assume that the statistics from the NCAA state that the average number of injuries per division one college baseball game is 1.9. What is the probability of at least two injuries in a single game? Round your answer to four digits past the decimal.

$$X \sim \text{Poi}(\lambda = 1.9) \quad P(X \geq 2) = 1 - P(X \leq 1) =$$

$$= 1 - [P(X=0) + P(X=1)] = 1 - \left[\frac{1.9^0 e^{-1.9}}{0!} + \frac{1.9^1 e^{-1.9}}{1!} \right] =$$

On IRS tax forms it is standard practice to round the listed amounts to the nearest dollar. This process ensures that the rounding error averages out to zero. Using cents as the unit, the rounding error would be a uniformly distributed random variable between the values of -50 and $+50$. Use this information to answer the next three questions.

0.24

magnitude?

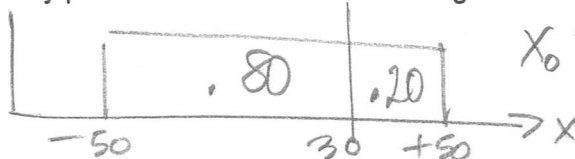
14. What is the probability that the rounding error is more than 38 cents in

$$P(X < -38) + P(X > 38) =$$

$$\frac{12}{100} + \frac{12}{100} = \frac{24}{100} = .24$$

30

15. Twenty percent of the time the rounding error exceeds what positive amount?



$$X_0 = 50 - .2(100) = 30$$

1

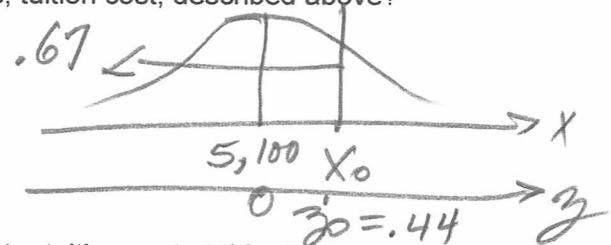
16. What is the probability that this variable has values that are within two standard deviations of the mean?

At a large state university, the in-state single semester tuition cost for a full-time student is normally distributed with a mean of \$5,100 and a standard deviation of \$125. Use this information to address the remaining questions on this page.

5,155

17. What is the 67th percentile for the variable, tuition cost, described above?

$$\begin{aligned} X_0 &= \mu + z_0 \sigma \\ &= 5,100 + (.44) 125 \\ &= 5,155 \end{aligned}$$



0.98354

18. What is the probability that a single semester tuition cost at this state university will between \$4,825 and \$5,450?

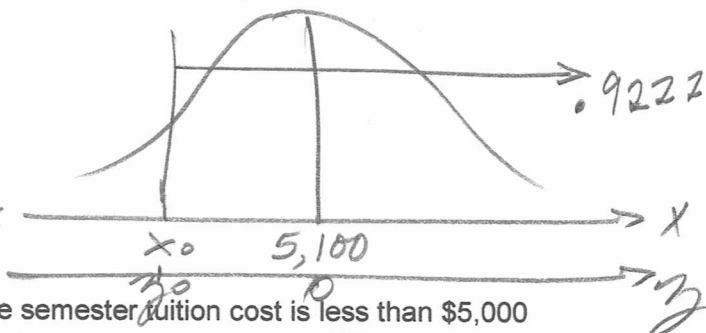
$$\begin{aligned} P(4,825 < X < 5,450) &= \\ &= P(-2.2 < Z < 2.8) = \end{aligned}$$

$$= .99744 - [1 - .98610] = 0.98354$$

19. The single semester tuition cost at this state university will be more than what amount 92.22 percent of the time?

$$z_0 = -1.42$$

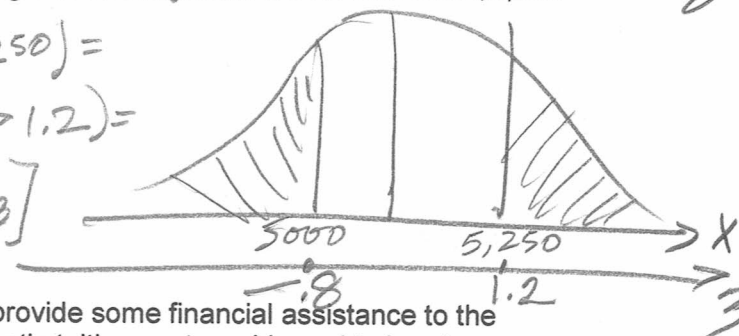
$$\begin{aligned} X_0 &= \mu + z_0 \sigma \\ &= 5,100 + (-1.42) 125 \\ &= 4,922.5 \end{aligned}$$



0.32693

20. What is the probability that a single semester tuition cost is less than \$5,000 or more than \$5,250?

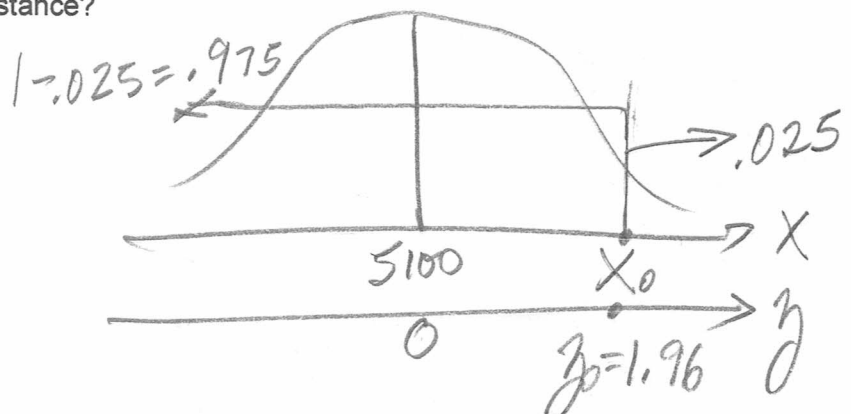
$$\begin{aligned} P(X < 5,000) + P(X > 5,250) &= \\ &= P(Z < -.8) + P(Z > 1.2) = \\ &= [1 - .78814] + [1 - .88493] \end{aligned}$$



5,345

21. If this state university wants to provide some financial assistance to the students whose tuition bills are in the top 2.5%, a student's tuition cost would need to be above amount in order to receive this financial assistance?

$$\begin{aligned} z_0 &= 1.96 \\ X_0 &= \mu + z_0 \sigma \\ &= 5,100 + (1.96) 125 \\ &= 5,345 \end{aligned}$$



One of the Chick-fil-a Chicken Sandwiches is advertised to have a mean calorie content of 440 calories. Assume the advertising is true and in addition, assume that the standard deviation of the calorie content of the sandwiches is 20 calories. Consider repeated random samples of 100 sandwiches and the resulting set of sample means. Use this sampling description to answer the questions on this page.

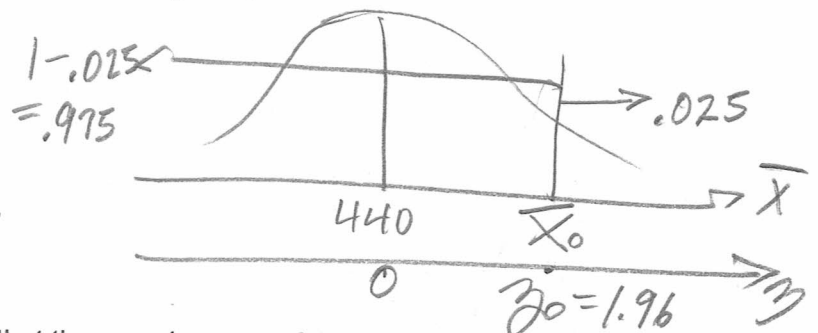
$$M = 440, \sigma = 20, n = 100$$

2 22. What is the typical spread, or standard deviation, of the individual sample means around the population mean of 440?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2$$

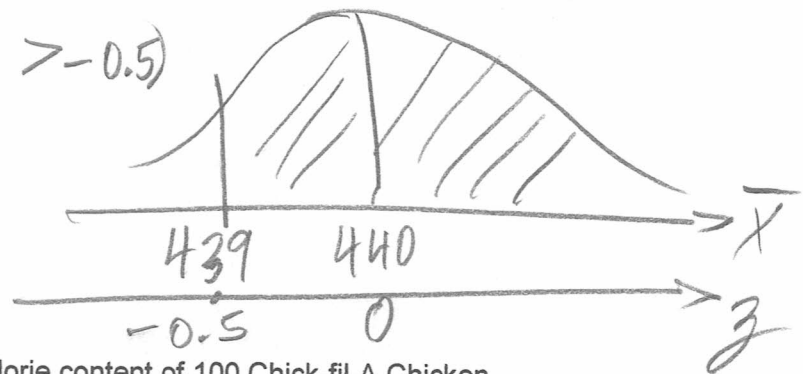
443.92 23. What is the value of the sample mean of the calorie content of 100 sandwiches that is exceeded by only 2.5% of all the resulting sample means?

$$\begin{aligned} z_0 &= 1.96 \text{ so,} \\ \bar{x}_0 &= M + z_0 \sigma_{\bar{x}} \\ &= 440 + (1.96)2 \\ &= 443.92 \end{aligned}$$



0.69146 24. What is the probability that the sample mean of the calorie content of 100 Chick-fil-a Chicken Sandwiches exceeds 439?

$$\begin{aligned} P(\bar{x} > 439) &= P(z > -0.5) \\ &= P(z < .5) = \\ &= 0.69146 \end{aligned}$$



437.64 25. The sample mean of the calorie content of 100 Chick-fil-A Chicken Sandwiches is less than what value 11.9% of the time?

$$\begin{aligned} z_0 &= -1.18 \\ \bar{x}_0 &= M + z_0 \sigma_{\bar{x}} \\ &= 440 + (-1.18)2 \\ &= 437.64 \end{aligned}$$

