STATISTICS 2023 | **NAME IN INK** Key

FINAL EXAM | **SIGNATURE IN INK**____________________

SPRING 1998 | **SS NUMBER IN INK**____________________

TRUE OR FALSE. Answer with a capital T or F. **(3 points each)**

T 1. Point estimates for parameters are always wrong and statistical procedures can be used to estimate their typical mistakes.

T 2. A confidence interval provides a reasonable or plausible set of values for the parameter being estimated.

T 3. A null hypothesis is probably false if the observed value of the test statistic is an unlikely value in the set of possible values which occur when the null hypothesis is true.

T 4. If a 95% confidence interval procedure is used to estimate a parameter there is a 5% chance that an interval will be generated that does not contain the true value of the parameter being estimated.

T 5. A point estimate for the parameter being estimated is used as the center of an interval estimate.

F 6. If a Z hypothesis test generates a test statistic value which is more than 5 in magnitude the null hypothesis would be not be rejected with any reasonable error rate.

F 7. If a small sample is used to test hypotheses about the population mean then the sample data must be assumed to be normally distributed.

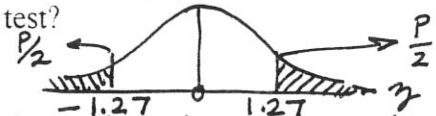
F 8. Statistical procedures can not be used to analyze qualitative variables which have words as the values of the variable.

F 9. If the p-value of a hypothesis test is almost one then the null hypothesis should definitely be rejected.

F 10. Point estimators are always wrong but interval estimators provide a span of values that always contains the true value of the parameter being estimated.

STATE THE ANSWER. State the answer on the line. **(3 points each)**

0.2040 11. If in a two-tail hypothesis test on the population mean the observed Z test statistic value is 1.27 what is the p-value of the test?



$\frac{P}{2} = .5 - .3980 = .1020$

12.6 12. If a sample with one-hundred observations gives a sum equal to 1260 and a sum of squares equal to 32,607 what is the value of the point estimate for the population mean?

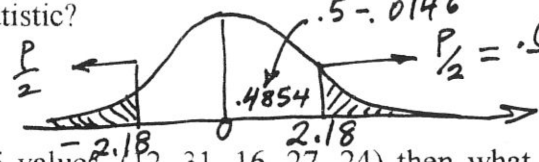
$$\bar{X} = \frac{\Sigma X}{n} = \frac{1260}{100} = 12.6$$

13 13. If a sample with one-hundred observations gives a sum equal to 1260 and a sum of squares equal to 32,607 what is the value of the point estimate for the population standard deviation?

$$S = \sqrt{S^2} = \sqrt{\frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n-1}} = \sqrt{\frac{32{,}607 - \frac{(1260)^2}{100}}{99}} = \sqrt{169} = 13$$

(can be + or −) 2.18 14. If the p-value in a two-tail hypothesis test based on a large sample is 0.0292 what is the magnitude of the test statistic?

TEST STAT is + or − 2.18
magnitude is 2.18.

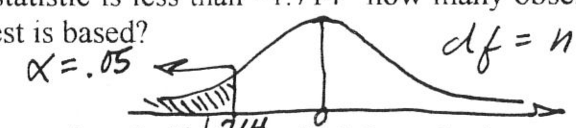


$P/2 = \frac{.0292}{2} = .0146$

61.5 15. If a sample has 5 values, (12, 31, 16, 27, 24) then what is the value of the sample variance?

$$S^2 = \frac{\Sigma(X-\bar{X})^2}{n-1} = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{n}}{n-1} = \frac{246}{4} = 61.5$$

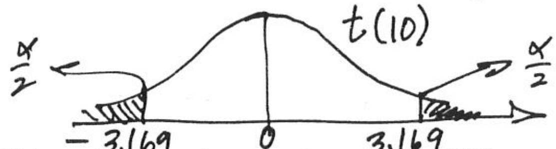
24 16. If the rejection rule for a 5% significance level in a hypothesis test on a small sample is, "Reject Ho if the test statistic is less than –1.714" how many observations are in the sample on which the hypothesis test is based?



$df = n - 1 = 23$

$\Rightarrow n = 24$

.01 17. If the rejection region for a two-tailed hypothesis test based on eleven observations is below –3.169 and above 3.169 what is the significance level of the hypothesis test?



$\frac{\alpha}{2} = .005 \Rightarrow \alpha = 2(.005) = .01$

11.5 18. If a confidence interval to estimate the difference between two populations means is (-12.4, 35.4) then what is the numerical value of the point estimate for the difference between the population means?

$\bar{X}$ is the center of the interval. $\frac{-12.4 + 35.4}{2} = 11.5$

1.58 19. If two samples with one-hundred observations each resulted in sample variances equal to 89 and 162 what is the numerical value of the standard error of the point estimate for the difference between the means of the populations from which these samples were drawn? Round your answer to two digits past the decimal. $n_1 = n_2 = 100$

$$S_{\bar{X}_1 - \bar{X}_2} = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{89}{100} + \frac{162}{100}} = 1.584297952$$

STATE THE ANSWER. State the answer on the line. **(3 points each)**

A tax reform group wants to compare the average annual property tax on houses in an urban setting with equally valued houses in a rural setting. Fifteen houses were randomly sampled from the urban tax role and fifteen houses were randomly sampled from the rural tax role. The average annual tax observed for urban and rural settings are given below along with the variances of each of the samples. Use this information to answer the questions on this page.

URBAN (sample ONE)	RURAL (sample TWO)
$\bar{x}_1 = \$235.00$	$\bar{x}_2 = \$242.50$
$s_1^2 = \$50.00$	$s_2^2 = \$85.00$

$\mu_1 < \mu_2$ $\mu_1 - \mu_2 < 0$ $\mu_2 - \mu_1 > 0$ 20. What is the appropriate alternative hypothesis if the research question is, "Do the data indicate that the average annual property tax for houses in an urban setting is less than the average annual property tax for houses in a rural setting?

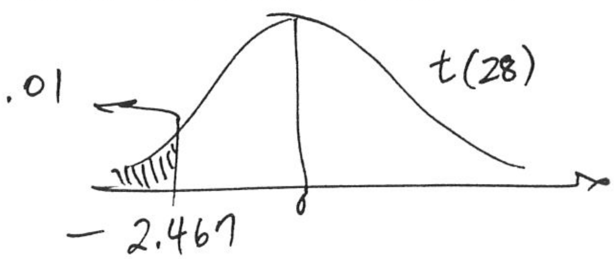
+ or − -2.5 21. What is the numerical value of the test statistic that would be used to attempt to support the alternative hypothesis described in the above question?

$$t = \frac{\bar{X}_1 - \bar{X}_2 - D_0}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} = \frac{235 - 242.5 - 0}{\sqrt{\frac{67.5}{15} + \frac{67.5}{15}}} = \frac{-7.5}{3} = -2.5$$

$t(28)$ 22. If the average annual property tax for houses is an urban setting is equal to the average annual property tax for houses is a rural setting what is the name of the distribution of the test statistic?

If H_0 is true then $t_{calc} \sim t(n_1 + n_2 - 2) \equiv t(28)$.

+ or − -2.467 23. In this situation the null hypothesis would be rejected at the 1% significance level if the observed test statistic value is less than what value? The answer is a negative number.



No 24. If the p-value of this hypothesis test is 0.08 would one conclude that the average annual property tax for houses in an urban setting is less than the average annual property tax for houses in a rural setting at a 5% significance level? Answer with YES or NO.

$P = .08 > .05 = \alpha \Rightarrow$ Do not Rej $H_0 \Rightarrow$ Do not Support H_A.

All the questions on this page are related to data set below. A bivariate data set had the following observed values. Use this data set to answer the questions on this page. **(2 points each)**

x	2	1	3	3	1
y	5	4	8	6	2

25. State the numerical values of the sufficient sums for the bivariate data given above.

$\Sigma x =$ 10 $\Sigma y =$ 25 $\Sigma x^2 =$ 24 $\Sigma y^2 =$ 145 $\Sigma xy =$ 58

26. State the numerical values of the three corrected sums of squares for the bivariate data given above.

$SS_{xx} =$ 4, $SS_{yy} =$ 20, $SS_{xy} =$ 8.

$SS_x = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 24 - \frac{10^2}{5} = 4.$ $SS_y = \Sigma y^2 - \frac{(\Sigma y)^2}{5} = 145 - \frac{25^2}{5} = 20.$

$SS_{xy} = \Sigma xy - \frac{\Sigma x \Sigma y}{n} = 58 - \frac{10(25)}{5} = 8.$

27. State the numerical value of the least squares estimates of the slope: 2

28. State the numerical value of the least squares estimate of the y-intercept: 1

The estimated regression equation for the bivariate data given above is: $\mathbf{\hat{y} = 1 + 2(x).}$
Use this equation to answer the remaining questions on this page. **(2 points each)**

5.4 29. What is the numerical value of the least squares estimate of the average y-value when x = 2.2?

$\hat{y}_{x=2.2} = 1 + 2(2.2) = 5.4$

1 30. How much does the least squares estimate of y increase if x increases by 0.5?

$\hat{\beta}_1(.5) = 2(.5) = 1.$

2.5 31. If the standard error of the least squares estimate of the slope is 0.80 what is the numerical value of the t-test statistic to test if the slope is equal to zero? If $S_{\hat{\beta}_1} = .80$

$t = \frac{\hat{\beta}_1}{S_{\hat{\beta}_1}} = \frac{2}{.8} = 2.5$

.89 32. What is the numerical value of the estimate of the linear correlation in this situation? Round your answer to two digits past the decimal.

$r = \frac{SS_{xy}}{\sqrt{SS_x \cdot SS_y}} = \frac{8}{\sqrt{4(20)}} = .8944272$