STATISTICS 2023 NAME, IN INK________________

FINAL EXAM SIGNATURE, IN INK________________

SPRING 1992 SS NUMBER, IN INK________________

If you do not want this exam returned publicly state that on the top.

TRUE OR FALSE. Answer with a capital T or F. (2 points each)

__T__ 1. The width of a confidence interval will decrease when the size of the sample increases if the level of confidence and sample variance remain the same.

__T__ 2. The point estimate for the population parameter is used as the center value in a confidence interval to estimate that population parameter.

__T__ 3. A 95% confidence interval to estimate a population parameter from a large sample is approximately four standard errors wide.

__T__ 4. If the p-value of a hypothesis test is 0.08 then the null hypothesis should be rejected for a maximum allowable error rate of 10%.

__F__ 5. The p-value in a hypothesis test is the error rate we must be willing to tolerate if we do not reject the null hypothesis.

__T__ 6. The p-value of a hypothesis test is the tail area associated with the test statistic.

__T__ 7. If the square of the sum of a data set with 100 observations is 160,000 then the mean of the data set is 4.00.

__F__ 8. The standard errors of point estimates increase in size as the sample size increases.

__F__ 9. For low degrees of freedom the t-distribution has less variance than the standard normal distribution.

__F__ 10. If a 95% confidence interval to estimate the population mean is (2.57, 3.86) then it should be concluded that 95% of the time the population mean is between 2.57 and 3.86.

__T__ 11. If the test statistic value in a Z-hypothesis test is 11 then the null hypothesis would be rejected for any significance level of 1% or less.

__F__ 12. The point estimates we use have good characteristics and are usually equal to the value of the parameter which they estimate.

__T__ 13. In a linear regression situation the intercept and the slope of the regression equation are the parameters.

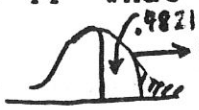
STATE THE ANSWER. State the answer on the line given.

(3 points each)

2.120 14. What is the value of the number which would be labelled as $t_{.025(16)}$?

0.01 15. If the rejection region in a two tail hypothesis test based on a large sample is below -2.576 and above 2.576 what is the maximum error rate of rejecting a true null hypothesis which this researcher will tolerate?

0.0179 16. If the value of a Z-test statistic is 2.1 and the alternative hypothesis reads "$\mu > 44$" what is the p-value or OSL of this hypothesis test?



0.02 17. In a two-tail hypothesis test on a population mean what is the p-value or OSL of the hypothesis test if the calculated test statistic based on eleven observations is 2.764 and the population variance is unknown?

69.13 18. If a 95% confidence interval based on a large sample to estimate a population mean is (427, 698) then what is the value of the standard error of the point estimate for the population mean? Round your answer to two digits past the decimal.

$\text{WIDTH} = 698 - 427 = 271 = 2B = 2[Z_{\alpha/2} S_{\bar{x}}] = 2[1.96 \cdot S_{\bar{x}}] \Rightarrow S_{\bar{x}} = \frac{271}{2(1.96)} = 69.1326$

The following bivariate data involving X = electrical cost and Y = overhead cost was used to estimate a regression line. The sufficient sums are shown. Use this data to answer questions 19-21.

x	25	73	53	71	27	36	33	46
y	58	184	128	159	64	84	91	103

$\Sigma x = 364$ $\Sigma y = 871$ $\Sigma x^2 = 19{,}034$ $\Sigma y^2 = 108{,}927$ $\Sigma xy = 45{,}448$

108.875 − 2.35

~~2.3~~ 2.35 19. State the numerical value of the least squares estimate of the slope of the regression line.

$\hat{\beta}_1 = 2.35335$

1.797 → 1.95 20. State the numerical value of the least squares estimate of the y-intercept for the regression line.

$\hat{\beta}_0 = 1.797228$

0.985 21. State the numerical value of the estimated linear correlation.

$r = 0.98548$

STATE THE ANSWER. State the answer on the line given.

(2 points each)

A marketing firm is interested in comparing two populations. The firm randomly sampled twenty-five observations from each of the populations. Use the results from the two samples to answer questions 22 - 28.

	Sample One	Sample Two
mean	1642	1367
variance	121	196

1,3~~8~~67 22. State the numerical value of the point estimate for the mean of population two.

$\bar{X}_2 = \hat{\mu}_2 = 1397$

2.8 23. State the numerical value of the estimated standard error of the point estimate for the mean of population two.

$S_{\bar{X}_2} = \frac{S_2}{\sqrt{n_2}} = \frac{\sqrt{196}}{\sqrt{25}} = \frac{14}{5} = 2.8$

11 24. State the numerical value of the point estimate for the standard deviation of population one.

$\hat{\sigma}_1 = S_1 = \sqrt{121} = 11$

275 25. State the numerical value of the point estimate for the difference between the means of populations one and two.

$\mu_1 - \mu_2 = \bar{X}_1 - \bar{X}_2 = 1642 - 1367 = 275$

158.5 26. State the numerical value of the pooled variance estimate which would result from these two samples.

$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = [24(121) + 24(196)]/48 = 158.5$

3.56 27. State the numerical value of the estimated standard error of the point estimate for the difference between the population means.

$S_{\bar{X}_1-\bar{X}_2} = \sqrt{S_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{158.5\left(\frac{1}{25}+\frac{1}{25}\right)} = \sqrt{12.68} = 3.5608$

X 1.62 28. State the numerical value of the test statistic to test equality of the two population variances.

$F = \frac{S^2_{BIG}}{S^2_{little}} = \frac{196}{121} = 1.619834$

Use the following sample of data to answer questions 29 - 34.

(26, 34, 12, 14, 29, 12, 22) (2 points each)

21.29 29. State the numerical value of the sample mean.

$\bar{X} = 21.2857$

3,641 30. State the numerical value of the sum of squares for this sample of data.

$\Sigma X^2 = X_1^2 + X_2^2 + \ldots + X_7^2 = 3{,}641$

22,201 31. State the numerical value of the square of the sum for this sample of data.

$(\Sigma X)^2 = (X_1 + X_2 + \ldots X_7)^2 = (149)^2 = 22{,}201$

78.24 32. State the numerical value of the sample variance.

$S^2 = 78.23809$

22 33. State the numerical value of the sample range.

Range = max − min = 34 − 12 = 22

22 34. State the numerical value of the sample median.

Rank Order. 12 12 14 22 26 29 34

MULTIPLE CHOICE. Answer with a capital letter. (3 points each)

C 35. If the range of a data set is 620 units then a good guess at the standard deviation of the data set is

A. 310 B. 124 C. 155 D. 620

$\hat{\sigma} = \frac{Range}{4} = \frac{620}{4} = 155$

C 36. If a sample of 49 observations produced a point estimate for the mean equal to 32 and a standard error for that estimate equal to 3.7, then the standard deviation of the sample is closest to

$\bar{x} = 32 \quad S_{\bar{x}} = 3.7 = \frac{s}{\sqrt{49}} = \frac{s}{7} =$

A. 82.0 B. 16.81 C. 25.9 D. 640.0

D 37. If a researcher has strong evidence against the null hypothesis in a hypothesis test the p-value of the test could be which of the following values?

A. 0.38 B. 0.98 C. 0.492 D. 0.003

A 38. Which one of the following null hypotheses should be addressed to test the proposition that the X variable in a regression situation does not affect the Y variable?

A. $H_0: \beta_1 = 0$ B. $H_0: \beta_1 = 1$

C. $H_0: \beta_1 = \beta_0$ D. $H_0: \beta_1$ = slope parameter

Assume that a regression line has been fitted to a bivariate data set containing 22 data values. The resulting least squares estimated regression equation appears below. Use this information to answer questions 39 - 42. (3 points each)

$$\hat{y} = 75.0 + 463.15\,(x)$$

28,790.3 39. What is the estimate of y when x is equal to 62?

$\hat{y}_{x=62} = 75.0 + 463.15(62) = 28,790.3$

463.15 40. For every one unit increase in x how much does the value of y increase?

75.0 41. State the numerical value of the least squares estimate of the y-intercept.

3.729 42. If the standard error of the least squares estimate of the slope is 124.2 what is the numerical value of the t-test statistic to test if the slope is equal to zero? Round your answer to two digits past the decimal.

$S_{\hat{\beta}_1} = 124.2$

64 $t = \frac{\hat{\beta}_1 - 0}{S_{\hat{\beta}_1}} = \frac{463.15 - 0}{124.2} = 3.729066023$