DISCUSSION SECTION NUMBER TO RETURN EXAM_____

STATISTICS 2023   NAME IN PRINT_____

EXAM THREE   SIGNATURE IN INK_____

SPRING 2002   ID OR SS IN INK _____

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TRUE OR FALSE. Answer with a capital T or F.   **(3 points each)**

____T____ 1. A confidence interval that estimates a population mean is centered on a point estimate for the population mean.

____F____ 2. Population parameters are used to estimate sample statistics.

____F____ 3. The standard error of the sample mean is equal to the sample mean minus the sample size.

____T____ 4. The magnitude of the standard error of the sample mean increases when the sample size decreases.

____F____ 5. In a hypothesis test the null hypothesis is rejected if the p-value of the test is large, since the p-value is the probability that the null hypothesis is false.

____F____ 6. In a hypothesis test the decision is whether the data supports or fails to support the hypothesized value of the population parameter stated in the null hypothesis.
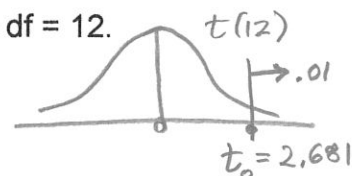
____F____ 7. The p-value in a hypothesis test is the tail area cut off by the rejection region.

Questions on the t-table. Write your answer on the line.   **(3 points questions)**

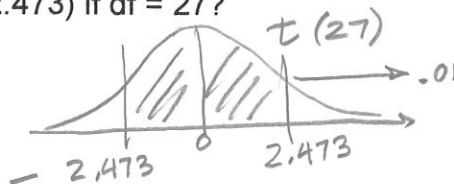____0.975____ 8. What is the $P(t < 2.110)$ if df = 17?

$= .5 + (.5 - .025) = .975$

$t(17)$
$.025$
2.110
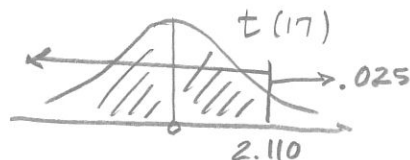
____2.681____ 9. State the value of t_o, if the $P(t > t_o) = .01$ and the df = 12.

$t(12)$
$.01$
$t_o = 2.681$

____0.98____ 10. What is the $P(-2.473 < t < 2.473)$ if df = 27?

$= 2(.5 - .01) =$
$= 2(.49) = .98$

$t(27)$
$.01$
-2.473 2.473

15.68 _____ **11.** How many units wide is a 95% confidence interval to estimate the mean based on a sample of 400 observations with a sample standard deviation of 80 units?

$$W = 2B = 2 \, z_{\alpha/2} \cdot S_{\bar{x}} = 2(1.96) \frac{80}{\sqrt{400}} = 15.68$$

7.5 _____ **12.** If the sum of the data in a sample with 250 observations is 1,875 what is the numerical value of the point estimate for the mean of the population from which the sample was drawn?

$$\hat{\mu} = \bar{X} = \frac{\Sigma X}{n} = \frac{1,875}{250} = 7.5$$

.0046 _____ **13.** If the researcher is addressing the question, "Do the data indicate that the mean is not equal to a certain value?" what is the p-value of the hypothesis test if the test statistic based on a large sample is equal to 2.83?

$$P/2 = .5 - .4977 = .0023 \Rightarrow P = 2(.0023) = .0046$$

2.201 _____ **14.** In a right tail hypothesis test on the mean based on a sample of only 12 observations what value must the test statistic exceed if the null hypothesis is rejected at the 2.5% significance level?

$t_{(11)}$ $\alpha = .025$ Reject H_0 at $\alpha = .025$ if
2.201 $t_{calc} > 2.201$.

115 _____ **15.** How many observations would be required to estimate the mean weight of standard production chickens at ten weeks of age to within .22 ounces with 95% confidence if the standard deviation of the weight of standard production chickens at ten weeks of age is known to be 1.2 ounces.

$$n \geq \frac{z_{\alpha/2}^2 \cdot \sigma^2}{B^2} = \frac{1.96^2 \cdot 1.2^2}{.22^2} = 114.29 \Rightarrow n \geq 115$$

.0002 _____ **16.** If in a right-tail hypothesis test based on a large sample of data the test statistic value is 3.5, then the p-value of the hypothesis test is what value?

$$P = P(Z > 3.5) = .5 - .4998 = .0002$$ 3.5

0.01 _____ **17.** In a hypothesis test based on twenty observations if the rejection rule in a left tail test is to reject the null hypothesis if the test statistic is less than -2.539, what is the significance level of the hypothesis test?

$$P(t_{(19)} < -2.539) = .01$$ α $t_{(19)}$ -2.539

.95 or 95% _____ **18.** If a sample of four hundred observations drawn from a population with a standard deviation 60 yields a confidence interval to estimate the population mean of (72.12, 83.88), what level of confidence was used to construct the interval?

$$W = 83.88 - 72.12 = 11.76 = 2B = 2 \, z_{\alpha/2} \cdot S_{\bar{x}} = 2 \, z_{\alpha/2} \left(\frac{60}{\sqrt{400}}\right) = 2 \, z_{\alpha/2}(3)$$

$$\Rightarrow 11.76 = 2 \, z_{\alpha/2}(3) \Rightarrow \frac{11.76}{2(3)} = z_{\alpha/2} = 1.96 \Rightarrow \alpha = .05 \Rightarrow Conf = .95$$

.36 _____ **19.** Assume a 95% confidence interval to estimate the proportion of students who commute to campus from outside Stillwater is (0.28, 0.44). What is the numerical value of the point estimate for the proportion of students who commute to campus from outside Stillwater?

$\hat{p}$ is the center of the interval (.28, .44).

(26.004, 26.396) _____ **20.** A sample of 400 red tail hawks had an average wingspan of 26.2 inches and a standard deviation of 2 inches. Based on this sample, what is the 95% confidence interval to estimate the average wingspan of the population of hawks from which this sample was drawn? Round the upper and lower bounds on your interval to three digits past the decimal.

$$26.2 \pm 1.96 \frac{2}{\sqrt{400}}$$

$$26.2 \pm .196 \Rightarrow (26.004, 26.396)$$

STATE THE ANSWER. Write the answer on the line.   (3 points each)

A sample of sixteen observations produced a mean of 175 and a standard deviation of 24. Use this information to answer the next four questions. $n=16, \bar{X}=175, S=24$

__175__ **21.** What is the numerical value of the point estimate for the mean of the population from which the above sample was drawn?

$$\hat{\mu} = \bar{X} = 175$$

__6__ **22.** What is the numerical value of the estimated standard error of the point estimate for the population mean based on the above sample information?

$$\hat{\sigma}_{\bar{x}} = S_{\bar{x}} = \frac{S}{\sqrt{n}} = \frac{24}{\sqrt{16}} = 6$$

__10.6 or 10.7__ **23.** If the standard error for the sample mean is 5 what is the numerical value of the bound of error for a 95% confidence interval to estimate the mean of the population from which the sample was drawn? State your answer with ~~two~~ one digits past the decimal.

$$B = t_{\frac{1}{2}(n-1)} \cdot S_{\bar{x}} = 2.131(5) = 10.655$$

__(165.4, 184.6)__ **24.** If the bound of error for a 95% confidence interval is 9.6 units what is the 95% confidence interval to estimate the population mean based on the sample mean stated above? State the interval.

$$\bar{X} \pm Bound$$
$$175 \pm 9.6 \implies (165.4, \ 184.6)$$

Eight hundred internet users were questioned about whether or not they had ever bid on an item on eBay, the online internet auction. Out of the eight hundred internet users questioned 280 of them responded that they had bid on at least one item in the eBay auction. Use this information to answer the remaining questions on this page.

__0.35__ **25.** Based on this sample what is the numerical value of the point estimate for the proportion of internet users who had bid on at least one item on eBay?

$$\hat{p} = \frac{X}{n} = \frac{280}{800} = 0.35$$

__.01686__ **26.** What is the numerical value of the estimated standard error for the point estimate for the proportion of internet users who had bid on at least one item on eBay? Round your answer to five digits past the decimal.

$$\hat{\sigma}_{\hat{p}} = S_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{.35(.65)}{800}} = .016863422$$

__10__ **27.** Assume the estimated standard error of the point estimate for the proportion of internet users who had bid on at least one item on eBay 0.005. What is the numerical value of the z test statistic to check if the proportion is equal to 30% against an alternative that the proportion is more than 30%?

$$\text{Assume } \hat{\sigma}_{\hat{p}} = .005 \qquad z = \frac{.35 - .30}{.005} = 10.0.$$

STATE THE ANSWER. Write the answer on the line. (3 points each)

The Internal Revenue Service (IRS) claims that the mean time to fill in a certain IRS form is 22 minutes. The times required by nine tax payers to fill in the specific IRS form were recorded and yielded a mean time of 37 minutes with a standard deviation of 15 minutes. Use this data as a random sample to answer the questions on this page.

_____ $M > 22$ _____ **28.** State the appropriate alternative hypothesis if the research question is, "Do the data provide evidence that the mean time required to complete the IRS form exceeds the stated time of 22 minutes?"

_____ 3 _____ **29.** What is the numerical value of the test statistic to test the null hypothesis that the average time required to complete the IRS form exceeds the stated time of 22 minutes?

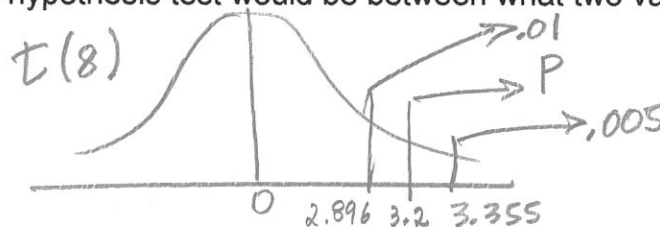$$t = \frac{37-22}{\frac{15}{\sqrt{9}}} = \frac{15}{5} = 3$$

_____ $t(8)$ _____ **30.** What is the name of the distribution of the test statistic if in fact the mean time required to fill in the IRS form is 22 minutes?

$$t(n-1) \equiv t(9-1) \equiv t(8)$$

_____ $.005, .01$ _____ **31.** If the numerical value of the test statistic in this case were 3.2 then the p-value of this hypothesis test would be between what two values?

_____ 2.896 _____ **32.** If the researcher performing this hypothesis test cannot tolerate more than 1% chance of rejecting a true null hypothesis then what value must the test statistic exceed in order to reject the null hypothesis?
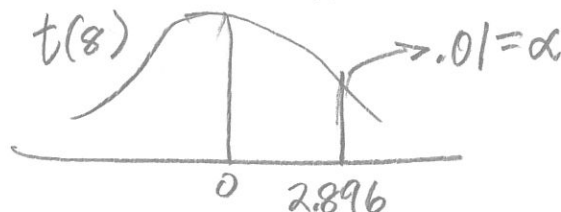
_____ Yes _____ **33.** If the p-value of this hypothesis test is between 0.025 and 0.010 and the significance level chosen by the researcher is 0.05 should the null hypothesis be rejected? Answer YES or NO.

$$.01 < P < .025 < .05 = \alpha \Rightarrow \text{Reject } H_0$$