STATISTICS 2023 NAME 2E2F91V1.CH1

EXAM TWO SIGNATURE _____

FALL 1991 SS NUMBER _____

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TRUE OR FALSE. Answer with a capital T or F. (3 points each)

T 1. In probability graphs for discrete random variables height represents the probability, but for continuous random variables the probability is represented by areas under the curves.

T 2. The possible values for a Binomial random variable are the whole numbers from zero to the number of trials in the experiment.

T 3. The standard normal distribution is a symmetric bell shaped distribution which can only have a mean of zero and standard deviation of one.

F 4. If a sample of size 36 is chosen from a population with mean of 25 and standard deviation of 6 then the probability that the resulting sample mean is less than 24 is 0.3413.

T 5. The standard error of a point estimator decreases in magnitude as the size of the sample on which the estimator is based gets larger.

F 6. The probability that the standard normal random variable has values between -1.25 and -0.50 is 0.5859.

T 7. The probability that the point estimate for a parameter is in a 90% confidence interval to estimate that parameter is one.

STATE THE SYMBOL. Write the correct symbol on the line. (3 points each)

s^2 8. Point estimator for the population variance

$s_{\bar{x}}$ or $\frac{s}{\sqrt{n}}$ 9. Estimator of the standard error of the sample mean

σ 10. Population standard deviation

$\bar{X}$ 11. Point estimator for the mean of the population

$\bar{X}$ 12. Sample mean

STATE THE ANSWER. State the answer on the line. (3 points each)

0.356 13. If X is a Binomial random variable with 8 trials in the experiment and the probability of success on any one trial is equal to 0.40 then what is the probability that the number of successes in the 8 trials is either 4 or 5?

$X \sim Bi(n = 8, p = .4)$

$P(X=4) + P(X=5) = .232 + .124 = 0.356$

0.077 14. If on the average 3.5 short-tail hawks can be sighted in a 2 hour time period in a certain rural location then what is the probability of seeing 6 short-tail hawks from that location in any 2 hour time period?

$X \sim Po(\mu = 3.5)$

$P(X=6) = 0.077$

0.205 15. Assume you are playing on a game show which has 10 possible doors to choose from and only 2 of those 10 doors are winners. If you get to open 5 of the doors during the game what is the chance that you will open both of the winning doors?

$X \sim Bi(n = 5, p = \frac{2}{10} = .2)$

$P(X=2) = 0.205$ Formula gives 0.2048.

74 16. If a sample of size 100 yields a 95% confidence interval to estimate the population mean which is (68, 80) what is the value of the mean of the sample of 100 observations?

$\bar{X}$ is the center of any C.I. for μ

0.9104 17. If a sample of size 64 is drawn from a population with a mean of 260 and a standard deviation of 24 then what is the probability that the resulting sample mean will be between 254 and 264.5?

$P(254 < \bar{X} < 264.5)$ $\quad \mu_{\bar{x}} = 260$

$= P\left(\frac{254-260}{3} < \frac{\bar{X}-\mu}{\sigma_{\bar{x}}} < \frac{264.5-260}{3}\right)$ $\quad \sigma_{\bar{x}} = \frac{24}{\sqrt{64}} = 3$

$= P(-2 < Z < 1.5) = .4772 + .4332 =$

(.352, .448) 18. Of 400 people questioned about whether they support the confirmation of Clarence Thomas to the supreme court only 160 stated that they did. Construct a 95% confidence interval based on this sample of 400 to estimate the proportion of people who support the Thomas confirmation. State the interval on the line. Round your answers to 3 digits past the decimal.

$\hat{p} = \frac{x}{n} = \frac{160}{400} = .40$

$\hat{p} \pm Z_{\frac{.05}{2}} \cdot S_{\hat{p}}$

$\hat{p} \pm Z_{.025} \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$.4 \pm 1.96 \sqrt{\frac{.4(.6)}{400}}$

$.4 \pm 0.048 \Rightarrow (0.35199, 0.448009)$

SHOW YOUR WORK. (6 points each)

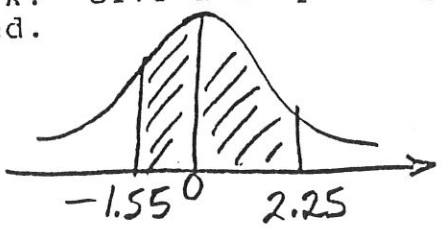
19. If 25% or one-fourth of the books in a person's private library are on the topic of gardening what is the probability that if you choose 6 books at random from this private library 3 of them will be on the topic of gardening? Give the basic equation. Show your work clearly.

$X \sim Bi(n=6, p=.25)$

$P(X=3) = C_3^6 \, .25^3 (.75)^3 =$

$= \frac{6!}{3!\,3!} \, .25^3 (.75)^3 = 0.131835 \simeq 0.13$

20. Assume that Z is the standard normal random variable. What is the probability that Z will be between 1.55 standard deviations below the mean and 2.25 standard deviations above the mean? Show all work. Give a complete graph with all parts of the problem labelled.



$P(-1.55 < Z < 2.25) =$

$= P(-1.55 < Z < 0) + P(0 < Z < 2.25) =$

$= .4394 + .4878 =$

$= 0.9272$

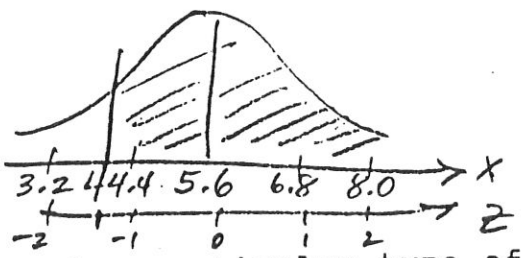
21. The amount of time students spend studying statistics is normally distributed with a mean of 5.6 hours per week and a standard deviation of 1.2 hours per week. If you spend 4 hours per week studying statistics what percent of students spend more time than you? Show all work. Give a complete graph with all parts of the problem labelled.

$X \sim N(M = 5.6, \sigma^2 = 1.2^2) \Rightarrow \sigma =$

$P(X > 4) =$

$= P\left(\frac{X-M}{\sigma} > \frac{4-5.6}{1.2}\right) =$

$= P(Z > -1.33) = .4082 + .5 = .9082$



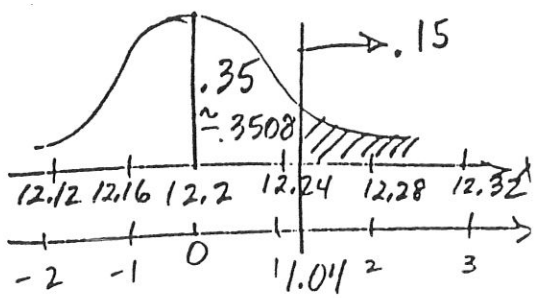
22. The amount of liquid placed in bottles of a particular type of soda pop is normally distributed with a mean of 12.2 ounces and a standard deviation of 0.04 ounces. How many ounces would a bottle contain if only 15% of all bottles contained more? Show all work. Give a complete graph with all parts of the problem labelled.

$X \sim N(M = 12.2, \sigma^2 = .04^2), \sigma = .04$

$X_0 = M + Z_0 \sigma$

$X_0 = 12.2 + (1.04).04$

$= 12.2416$ oz



SHOW YOUR WORK. (6 points each)

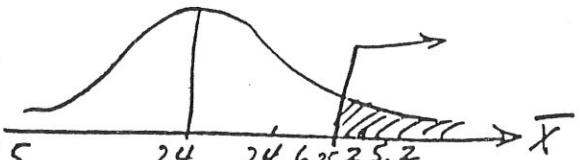
23. Assume the population of western red cedar trees in Oklahoma has a mean of 24 feet and a standard deviation of 6 feet. If a sample of 100 western red cedar trees is drawn what is the probability that the average height in the sample is more than 25 feet? Show all work. Give a complete graph with all parts of the problem labelled. $X \sim ?\ (\mu = 24,\ \sigma^2 = 6^2),\ \sigma = 6,\ n = 100$

$\mu_{\bar{x}} = \mu = 24,\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{100}} = .6$

$P(\bar{X} > 25) =$

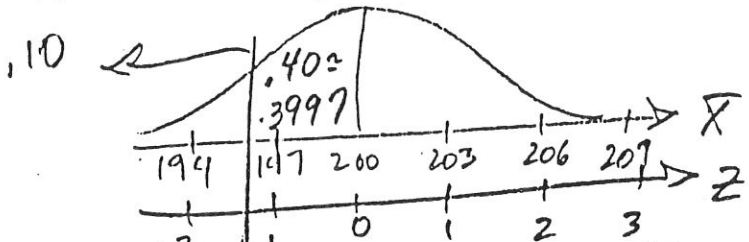
$P\left(\frac{\bar{X}-\mu}{\sigma_{\bar{x}}} > \frac{25-24}{.6}\right) =$ or $1.66 \Rightarrow .0485$

$P(Z > 1.67) = .5 - .4525 = .0475$



24. Students spend on the average 200 dollars per month on food with a standard deviation of 36 dollars. If samples each comprised of 144 students were drawn 10% of the resulting sample means would be less than what value? Show all work. Give a complete graph with all parts of the problem labelled.

$X \sim ?(\mu = 200, \sigma^2 = 36^2)\ \sigma = 6,\ n = 144,\ \mu_{\bar{x}} = \mu = 200,\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{36}{\sqrt{144}} = 3$



$\bar{X}_0 = \mu + Z_0 \sigma_{\bar{x}}$

$= 200 + (-1.28)3$

$= 196.16$

25. If a sample of 500 observations yielded a mean of 453 and a standard deviation of 42 then what is a 98% confidence interval to estimate the mean of the population from which this sample is drawn? Give the basic equation. Show all work. State the interval with the values rounded to two digits past the decimal.

$\bar{X} = 453,\ S = 42,\ n = 500$

$\bar{X} \pm Z_{\frac{.02}{2}} \cdot \frac{S}{\sqrt{n}}$

$453 \pm 2.326 \frac{42}{\sqrt{500}}$

$453 \pm 4.36892 \Rightarrow (448.63,\ 457.37)$

26. Twenty-two Canadian geese were sampled and their wing spans were recorded. The average wing span based on this sample of 22 geese was 40.4 ~~inches~~ with a standard deviation 2.2 inches. Use this sample to construct a 90% confidence interval to estimate the average wing span of Canadian geese. Give the basic equation. Show all work. State the interval with the values rounded to two digits past the decimal. $n = 22,\ \bar{X} = 40.4,\ S = 2.2$

$\bar{X} \pm t_{\frac{.10}{2}(21)} \frac{S}{\sqrt{n}}$

$40.4 \pm 1.721 \frac{2.2}{\sqrt{22}}$

$40.4 \pm 0.80722 \Rightarrow (39.59,\ 41.21)$