

DISCUSSION SECTION NUMBER FOR EXAM RETURN _____

STATISTICS 2023

NAME, IN INK _____

EXAM TWO

SIGNATURE, IN INK _____

FALL 2002

SS NUMBER, IN INK _____

Retain this exam for grade verification once it is graded and returned to you.

TRUE OR FALSE. Answer with a capital T or F.

(4 points each)

T 1. The number of people in a passenger vehicle is a discrete random variable.

F 2. The Binomial random variable has 10 values regardless of the number of trials or the probability of success on one trial.

F 3. Probability density functions are graphs of probability spikes associated with values of a discrete random variable.

F 4. The expected value of a variable is the value of the variable that has the greatest probability of occurring.

T 5. A uniform continuous random variable has constant probability over a specific range of possible values.

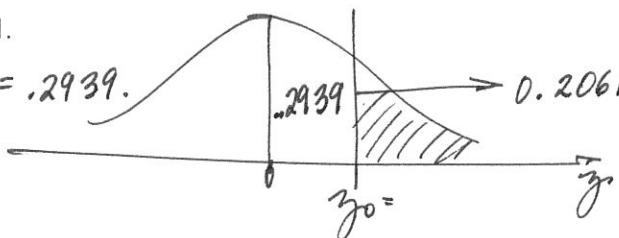
STANDARD NORMAL DISTRIBUTION QUESTIONS. State the answer on the line provided.

(4 points each)

0.82 6. Find z_0 if $P(Z > z_0) = 0.2061$.

$$P(0 < Z < z_0) = .5 - .2061 = .2939$$

$$\Rightarrow z_0 = 0.82$$

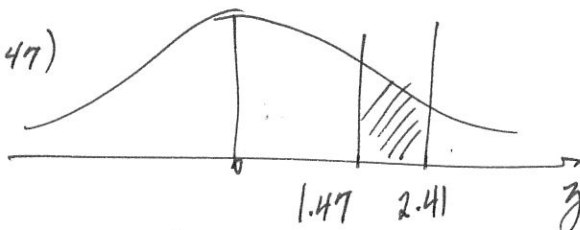


.0628 7. Find the $P(1.47 < Z < 2.41)$.

$$= P(0 < Z < 2.41) - P(0 < Z < 1.47)$$

$$= 0.4920 - 0.4292$$

$$=$$

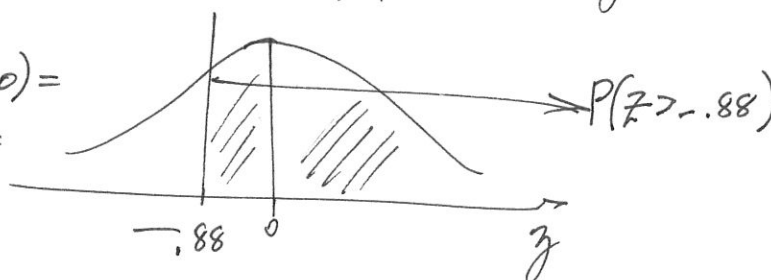


0.8106 8. What is the $P(Z > -0.88)$?

$$= P(-.88 < Z < 0) + P(Z > 0) =$$

$$= 0.3106 + .5 =$$

$$= 0.8106$$



STATE THE ANSWER. State the answer on the line given.

(4 points each)

\$8.00

9. Consider a lottery game in which a person can win \$0, or \$8,000. If only one person out of 1,000 people who play the lottery game win the \$8,000 prize what is the expected winnings in such a lottery game?

X	$P(X)$
0	$1 - \frac{1}{1000}$
8,000	$\frac{1}{1000}$

$$EX = M = \sum Xp(x) = 0\left(1 - \frac{1}{1000}\right) + 8000\left(\frac{1}{1000}\right) = \$8.00$$

.4

10. Assume that a discrete random variable has the values of 5, 10, 15, 20, 25, and 30. Further assume that the value 30 has probability of 0.50 associated with it, but each of the other values is equally likely. What is the probability that such a random variable has the value of 20 or less?

X	5	10	15	20	25	30
$P(X)$	.1	.1	.1	.1	.1	.5

$$P(X \leq 20) = .1 + .1 + .1 + .1 = .4$$

0.0444

11. Five percent of the cars returned to a car rental agency need the carpet shampooed prior to being rented again. If seven cars are rented what is the probability that at least two of them will need the carpet shampooed prior to being rented again? State your answer with four digits past the decimal.

$$X \sim Bi(n=7, p=0.05), X = \# \text{ of cars that need to be shampooed.}$$

$$P(X \geq 2) = P(X=2) + P(X=3) + \dots + P(X=7) \text{ OR } 1 - P(X \leq 1) = 1 - .9556 = .0444.$$

0.18208

12. Assume that eight percent of web pages on the internet will not load correctly when requested. If you attempt to load ten pages from the internet onto your computer what is the probability that two or three of them will not load correctly. State your answer with five or more digits past the decimal.

$$X \sim Bi(n=10, p=0.08), X = \# \text{ of pages that will not load correctly.}$$

$$\begin{aligned} P(X=2 \text{ or } 3) &= P(X=2) + P(X=3) \\ &= \binom{10}{2} .08^2 .92^8 + \binom{10}{3} .08^3 .92^7 \\ &= 0.147807035 + 0.034274095 \\ &= 0.18208113. \end{aligned}$$

0.4695

13. If the average number of DUI arrests in Stillwater each night of the weekend is 2.8 what is the probability of fewer than ~~four~~ ³ DUI arrests on one weekend night? State your answer with ~~four~~ ^{four} digits past the decimal.

$$X \sim Poi(\lambda=2.8), X = \# \text{ of DUI each night}$$

$$\begin{aligned} P(X < 3) &= P(X=0) + P(X=1) + P(X=2) \\ &= 0.0608 + 0.1703 + 0.2384 \\ &= 0.4695. \end{aligned}$$

0.27764

14. If on average there are 4.2 emergencies per day that arrive at a large veterinarian hospital what is probability that in one day there would be five or six emergencies? State your answer with five or more digits past the decimal.

$$X \sim Poi(\lambda=4.2), X = \# \text{ of emergencies per day}$$

$$\begin{aligned} P(X=5 \text{ or } 6) &= P(X=5) + P(X=6) \\ &= \frac{4.2^5 e^{-4.2}}{5!} + \frac{4.2^6 e^{-4.2}}{6!} \\ &= 0.163315867 + 0.114321107 \\ &= 0.277636974. \end{aligned}$$

OR
0.27763

STATE THE ANSWER. State the answer on the line given.

(4 points each)

The amount of sand needed by the University for sanding the sidewalks and the parking lots after a winter ice storm is uniformly distributed between the values of 8 tons and 16 tons. Use this information to answer the next three questions.

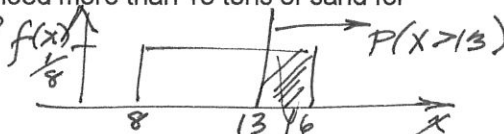
$$X \sim \text{UnifCont}(8, 16)$$

12 15. What is the expected amount of sand needed by the University for sanding the sidewalks and parking lots after a winter ice storm?

$$\frac{3}{8} \quad EX = \mu = \frac{c+d}{2} = \frac{8+16}{2} = 12 \text{ tons}$$

3/8 16. What is the probability that the University will need more than 13 tons of sand for sanding the sidewalks and parking lots after a winter ice storm?

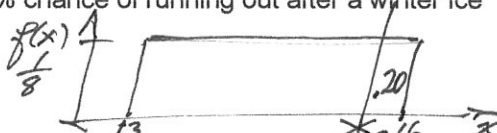
$$P(X > 13) = (16-13) \cdot \frac{1}{8} = \frac{3}{8}$$



14.4 17. In order to be prepared for ice storm management the University stock piles sand during the winter months for sanding the sidewalks and the parking lots. How many tons of sand should the University have in stock if the Physical Plant only wants a 20% chance of running out after a winter ice storm?

$$Bh = .2$$

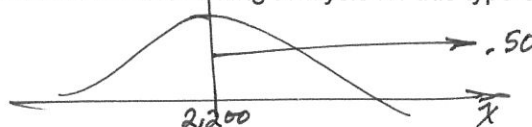
$$(16-x_0) \cdot \frac{1}{8} = .2 \Rightarrow x_0 = 14.4$$



The cost of an accounting analysis for a certain type of small business is a normally distributed random variable with a mean of \$2,200 and a standard deviation of \$175. Use this information to answer the next four questions.

$$X \sim N(M = \$2,200, \sigma^2 = 175^2), \sigma = \$175$$

2,200 18. Half of the time the cost of the accounting analysis for this type of small business exceeds what value?



0.6447 19. If the distribution of the cost is as described above what is the probability that the cost of this type of accounting analysis is between \$2,130 and \$2,602.50?

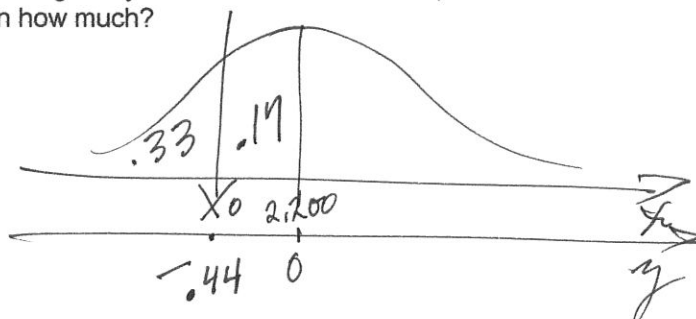
$$\begin{aligned} P(2,130 < X < 2,602.50) &= \\ &= P\left(\frac{2,130-2,200}{175} < \frac{X-M}{\sigma} < \frac{2,602.50-2,200}{175}\right) \\ &= P(-.4 < Z < 2.3) = \end{aligned}$$

0.1561 20. If the distribution of the cost of the accounting analysis is as described above what is the probability of the cost exceeding \$2,375 but being less than \$2,690?

$$\begin{aligned} P(2,375 < X < 2,690) &= \\ &= P\left(\frac{2,375-2,200}{175} < \frac{X-M}{\sigma} < \frac{2,690-2,200}{175}\right) = \\ &= P(1 < Z < 2.8) = P(0 < Z < 2.8) - P(0 < Z < 1) = .4974 - .3413 = \\ &= .1561. \end{aligned}$$

2,123 21. If the distribution of the cost of the accounting analysis is as described above, then 33% of the time the cost of the accounting analysis is less than how much?

$$\begin{aligned} X_0 &= M + z_0 \sigma \\ &= 2,200 + (-.44)175 \\ &= 2,123. \end{aligned}$$



STATE THE ANSWER. State the answer on the line given.

(4 points each)

The life of a car battery is measured in months rather than in miles that the car is driven. A certain type of car battery has a mean lifetime of 48 months with a standard deviation of 5 months. Assume that random samples of size one-hundred were repeatedly drawn from batteries of this type and the lifetimes in months were recorded. Use this information to answer the questions on this page.

48 22. What is the numerical value of the mean of all possible sample means that would result from repeated samples of one-hundred car battery lifetimes?

From above $M = 48$, $\sigma = 5$.

$$\bar{X} \sim N(M_{\bar{X}} = M = 48, \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n} = \frac{5^2}{100}), \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{100}} = .5$$

0.5 23. What is the numerical value of the standard deviation of all possible sample means that would result from repeated samples of one-hundred car battery lifetimes?

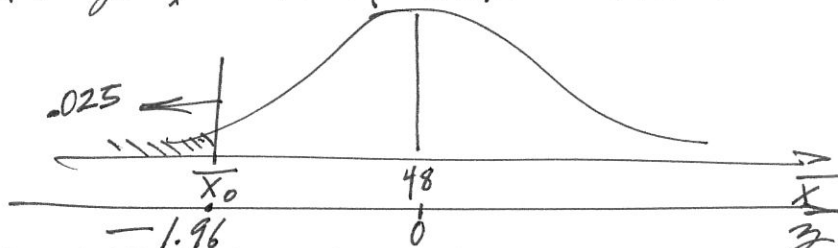
$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{100}} = 0.5$$

47.02 24. Only 2.5% of the sample means of car battery lifetimes that result from the above repeated sampling situation will be less than what value?

$$\text{Find } \bar{X}_0, P(\bar{X} < \bar{X}_0) = 0.025$$

$$\text{Find } z_0, P(Z < z_0) = 0.025 \Rightarrow z_0 = -1.96$$

$$\bar{X}_0 = M + z_0 \sigma_{\bar{X}} = 48 + (-1.96) \cdot .5 = 47.02$$



0.8185 25. What is the probability that the sample means that result from repeated sampling of the car battery lifetimes will be between 47.5 and 49 months?

$$P(47.5 < \bar{X} < 49) =$$

$$= P\left(\frac{47.5 - 48}{.5} < \frac{\bar{X} - M}{\sigma_{\bar{X}}} < \frac{49 - 48}{.5}\right)$$

$$= P(-1 < Z < 2)$$

$$= P(-1 < Z < 0) + P(0 < Z < 2) =$$

$$= .3413 + .4772$$

$$= 0.8185.$$

