STATISTICS 2023 **NAME, PRINT IN INK**____________________

EXAM TWO **SIGNATURE, IN INK**____________________

SPRING 2017 **CWID, IN INK**____________________

Once this exam is graded and returned to you retain it for grade verification.

TRUE OR FALSE. Answer with a capital T or F. **4 points each)**

F 1. A discrete random variable is a variable that only has probability on intervals of values and no probability at all on specific values.

F 2. A probability density function is a function that indicates how much of the mass of the unit probability is assigned to each value of a discrete random variable.

T 3. The standard normal distribution is a symmetric, bell-shaped distribution, which always has a standard deviation equal to one and a mean equal to zero.

T 4. The Central Limit Theorem states that regardless of the shape of the original population, if samples of adequate size are repeated drawn, then the resulting sample mean values are approximately normally distributed.

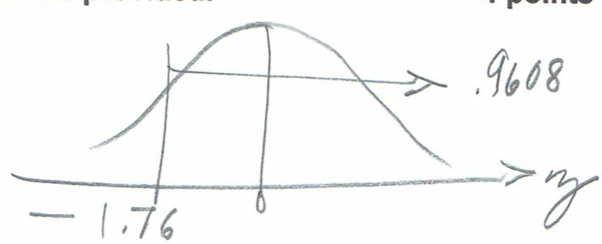
T 5. The mean of the sample means, which result from repeated sampling, is always equal to the mean of the population from which the random samples were drawn.

STANDARD NORMAL DISTRIBUTION QUESTIONS.
STATE THE ANSWER State the answer on the line provided. **4 points each)**

-1.76 6. Find z_o, if $P(Z > z_o) = 0.9608$.

Z - table

	.06
1.7	.9608

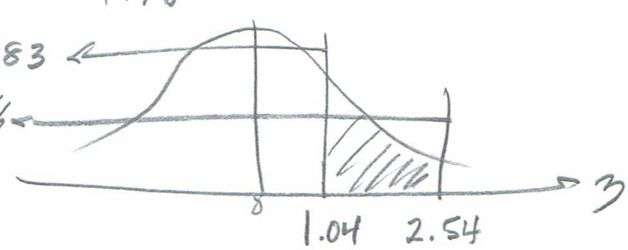


0.14363 7. Find the $P(1.04 < Z < 2.54)$.

$= P(z < 2.54) - P(z < 1.04)$

$= .99446 - .85083 =$

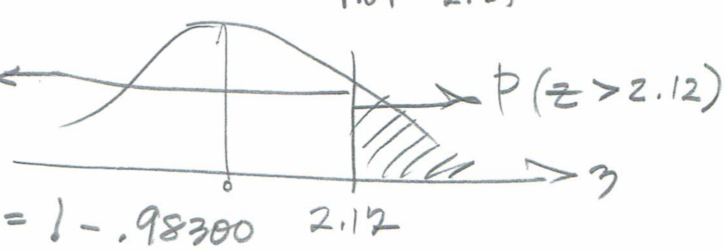
$= 0.14363$



0.017 8. What is the $P(Z > 2.12)$?

$P(z > 2{,}12) =$

$1 - P(z < 2{,}12) = 1 - .98300$

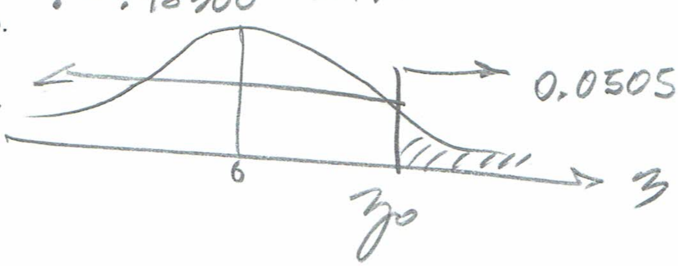


$1{,}64$ 9. Find z_o, if $P(Z > z_o) = 0.0505$.

z table

$1 - 0.0505 =$
$= .9495$

	.04
1.6	0.9495



STATE THE ANSWER. State the answer on the line give **(4 points each)**

\$123.75 10. In an investment that has 0.50 probability of making \$100, 0.25 probability of making \$125, 0.15 probability of making \$150, and the remaining probability on making \$200, what is the expected earning from this investment?

X	100	125	150	200
$P(x)$	0.50	0.25	0.15	0.10

$\mu = \sum x p(x) =$

$.5(100) + .25(125) + .15(150) + .10(200) = = \$123.75$

0.134 11. A gambler has the probability 0.42 to win a game. If this game is played seven times what is the probability that the gambler will win fewer than 2 times? Round your answer to three digits past the decimal. $X = \#$ of wins out of 7. $X \sim Bi(7, 0.42)$

$$P(X<2) = P(X \le 1) = P(X=0) + P(X=1) = \binom{7}{0}.42^0 .58^7 + \binom{7}{1}.42^1 .58^6$$

$$= 0.13400179$$

13 12. Sixty-Five percent of all households in the US own more than 2 cars. Suppose twenty households have been randomly selected. What is the expected number of households with more than 2 cars out of the 20 sampled? $X = \#$ of households w/ more than 2 cars

$X \sim Bi(n = 20, p = .65)$ so

$$EX = \mu = np = 20(.65) = 13$$

0.7306 13. On average, 1.8 customers enter the bookstore in Student Union every minute. What is the probability that at most two customers will enter the bookstore in one minute? State the answer with four digits past the decimal. $X = \#$ of customers who enter the store in one minute.

$X \sim Poi(\lambda = 1.8)$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2) = \frac{1.8^0 e^{-1.8}}{0!} + \frac{1.8^1 e^{-1.8}}{1!} + \frac{1.8^2 e^{-1.8}}{2!} = 0.7306$$

Suppose the research department of a steel manufacturer is concerned that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness is a uniform continuous random variable with values between 140 and 200 millimeters. Use this information to answer the remaining questions on this page. $X \sim UnfCont(140, 200)$

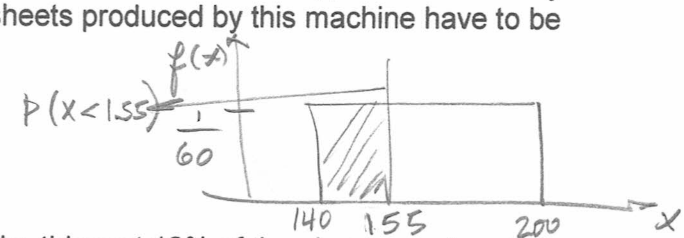
170 14. What is the mean thickness of the sheets produced by this machine?

$$\mu = \frac{c+d}{2} = \frac{140+200}{2} = 170$$

$\frac{15}{60}$ or .25 15. Sheets with thickness less than 155 millimeters must be scrapped because they are unacceptable to buyers. What is the probability that sheets produced by this machine have to be scrapped?

$P(X<155) = Bh$

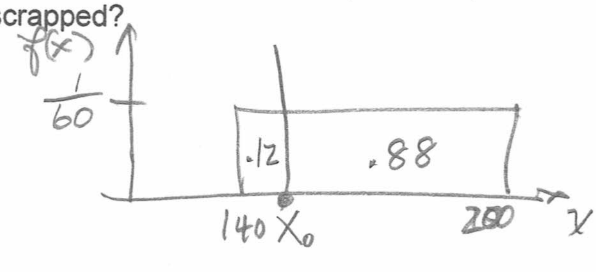
$= (155-140)\frac{1}{60} = \frac{15}{60} = 0.25$



147.2 16. If the company only wants to scrap the thinnest 12% of the sheets produced, then the sheets thinner than how many millimeters should be scrapped?

$X = 140 + .12(60)$ OR

$X = 200 - .88(60) = 147.2$



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STATE THE ANSWER. State the answer on the line given. **(4 points each)**

A new wheat cultivar produces a yield under ideal conditions that is normally distributed with a mean of 62 bushels per acre with a standard deviation of 18 bushels per acre. Use this information to answer the questions on this page. X= wheat yield, $X \sim N(62, 18^2)$, $\sigma = 18$

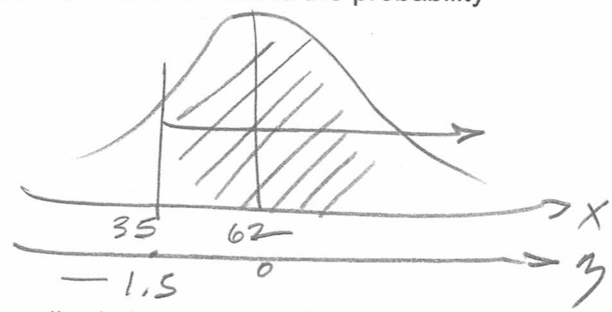
0.93319 17. Based on the distribution of wheat yield described above what is the probability that the yield will exceed 35 bushels per acre?

$P(X > 35) =$

$= P\left(\frac{X-\mu}{\sigma} > \frac{35-62}{18}\right)$

$= P(Z > -1.5) =$

$= P(Z < 1.5) = 0.93319$



97.28 18. Based on the distribution of wheat yield described above what is the value of the yield that is only exceeded by 0.025 of the yields?

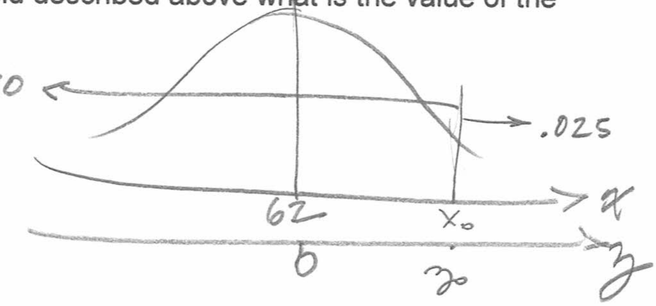
$P(X > x_0) = .025$

1. $P(Z > z_0) = .025 \Rightarrow z_0 = 1.96$

2. $x_0 = \mu + z_0 \sigma = 62 + 1.96(18) =$

$= 97.28$

$1 - .025 = .9750$



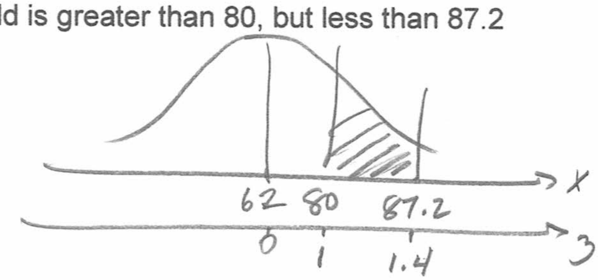
0.0779 19. What is the probability that the wheat yield is greater than 80, but less than 87.2 bushels per acre? $P(80 < X < 87.2) =$

$= P\left(\frac{80-62}{18} < \frac{X-\mu}{\sigma} < \frac{87.2-62}{18}\right) =$

$= P(1 < Z < 1.4) =$

$= P(Z < 1.4) - P(Z < 1) =$

$= 0.91924 - .84134 = 0.0779$



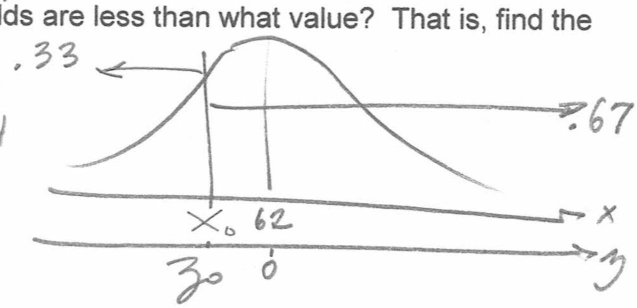
54.08 20. Thirty-three percent of the wheat yields are less than what value? That is, find the 33rd percentile of the wheat yields.

$P(X < x_0) = .33$

1. $P(Z < z_0) = .33 \Rightarrow z_0 = -.44$

2. $x_0 = \mu + z_0 \sigma = 62 + (-.44)18$

$= 54.08$



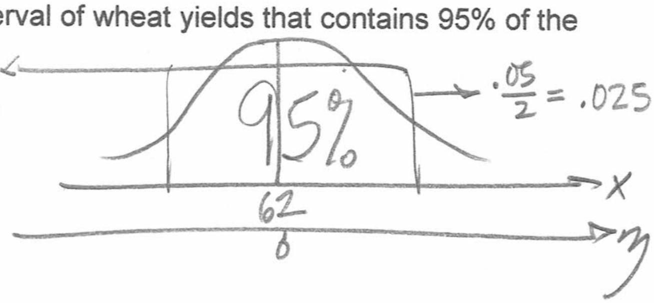
(26.72, 97.28) 21. What is the interval of wheat yields that contains 95% of the wheat yields and is centered on the mean?

.06

1.9 .9750

$1 - .025 =$

$= .9750$



Interval is $\mu \pm 1.96\sigma$

$62 \pm 1.96(18)$

(26.72, 97.28)

STATE THE ANSWER. State the answer on the line given. **(4 points each)**

Assume 100 observations were randomly drawn from a population of investment returns with a mean of 1,650 dollars and a standard deviation of 50 dollars. Use this information to answer the remaining questions.

1650 22. What is the numerical value of the mean of all possible sample means that would result from the above situation?

$$\mu_{\bar{x}} = \mu = 1{,}650$$

5 23. What is the numerical value of the standard deviation of all possible sample means that would result from the above situation?

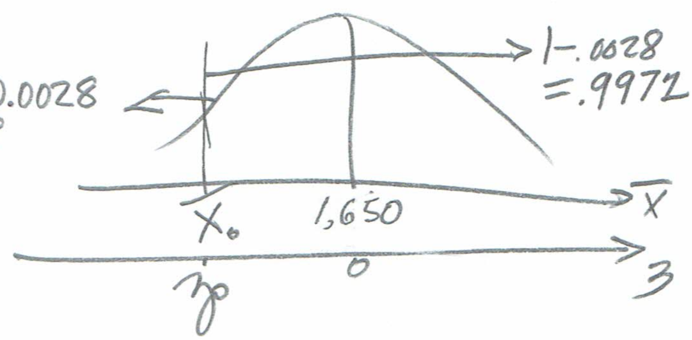
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{\sqrt{100}} = 5$$

1636.15 24. Only 0.28% (or 0.0028) of the sample means that result from the above sampling situation will be less than what value?

Find $\bar{x}_0$, $P(\bar{x} < \bar{x}_0) = .0028$

1. Find z_0, $P(z < z_0) = .0028$

$\Rightarrow z_0 = -2.77$



0.64499 25. What is the probability that the sample mean that results from the above situation will be between 1,647 and 1,657 dollars?

$$P(1{,}647 < \bar{x} < 1657) =$$

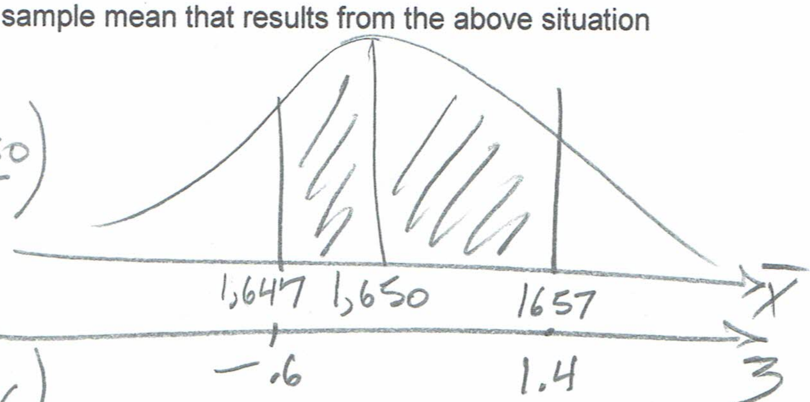
$$= P\left(\frac{1647-1650}{5} < \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} < \frac{1657-1650}{5}\right)$$

$$= P(-.6 < z < 1.4) =$$

$$= P(z < 1.4) - P(z < .6)$$

$$= .91924 - (1 - .72575) =$$

$$= 0.64499$$



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997