Lesson 8
Binomial Random Variables

Learning Objectives

Upon completion of this lesson you should be able to do the following:

1. Recognize the situation and variable description that generates a Binomial random variable.

2. Use the Binomial probability mass function to calculate probability for values of the Binomial random variable.

3. Calculate the expected value and variance for a Binomial random variable.

4. Graph the probability distribution for a Binomial random variable.

5. Calculate binomial probabilities with the Binomial table.

Key Words

binomial distribution, trial, outcomes, success, failure, binomial random variable

Concepts

The past lesson introduced the concept of random variables and their probability distributions. This lesson covers the ideas associated with a specific discrete random variable called the Binomial random variable. Different kinds of discrete probability distributions arise when specific variables are measured in certain conditions. What is the Binomial random variable and under what conditions does it occur?

Binomial Random Variable

The **binomial distribution** arises from an experiment that consists of n independent trials, where each **trial** is defined to have two possible **outcomes**. These two outcomes are called **success** and **failure**. The **binomial random variable** the number of success outcomes out of n independent trials. The possible values of the binomial random variable are integer values from zero to n. The probability of the outcome called success is designated with the letter p. The value of p, the probability of success on one trial, is constant from trial to trial since the trials are independent. The probability of failure on one trial is q, where $q+p=1$.

The probability distribution associated with the binomial random variable is used to calculate the probability of x-number of success outcomes out of n independent trials. To approach any binomial problem first recognize from the situation how many trials are involved, that is the n value. Then decipher the probability on any one trial of the outcome called success, that is the p value.

The parameters of the binomial probability distribution are the values of n and p. The values of n and p identify the specific binomial distribution for the situation. The expected value or mean of the binomial random variable and the variance are functions of n and p. The mean of the binomial random variable is np and the variance of the binomial random variable is npq. The quantities, np and npq, are the same values as the general equations learned earlier for the mean and variance of a discrete random variable.

To write that X has a Binomial distribution with parameters n and p use the symbols $X \sim Bi(n,p)$. Read as, "The variable X is distributed according to the Binomial distribution with parameters n and p."

Binomial Random Variable with parameters n and p: $X \sim Bi(n,p)$

The Variable: $X = \textit{the number of success outcomes in n independent trials}$

Possible Values of the Variable: $x = 0, 1, 2, \ldots, n.$

Binomial Probability Mass Function:

$$P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!(n-x)!} p^x q^{n-x},\ x = 0, 1, 2, \ldots, n.$$

Where $p = \textit{probability of success on one trial and } q = 1 - p,$

$n = \textit{number of independent trials.}$

Expected Value of X : $\mu = EX = \sum xP(x) = np.$

Variance of X : $\sigma^2 = E(X-\mu)^2 = \sum (x-\mu)^2 P(x) = \sum x^2 P(x) - \mu^2 = npq.$

Exercise with Answers

Binomial Random Variable

The Oklahoma State University Basketball team has a 70% chance of beating its in-state rival univeristy in each game the two teams play. Assume the two teams play three games during teh season. The variable of interest is $X = \textit{number of wins for OSU in the three games with the rival university.}$

1. What is the name of the probability distribution that describes the probability of such a variable?
2. State the values of the variable.
3. Calculate the probability for each value of the variable.
4. Present the probability distribution in a table form. Include the cumulative probability in an extra column. Notice the Binomial Table has the probability entries for the parameter values of n=3 and p=0.70.
5. Present the probability distribution in a graph form.
6. What is the expected number of wins for OSU from of the three games played?
7. What is the chance of OSU winning at least two of the games?
8. Write a sentence to be publisned in the O'Collegian concerning the probabililty calculated in number 7?

Answers

1. The name of the probability distribution for the variable, number of wins out of three games, is the Binomial distribution with parameter values of n=3 and p=0.70.

2. The values of the variable are $x = 0, 1, 2, 3.$

3. $P(X=0) = \binom{3}{0}.7^0.3^{3-0} = \frac{3!}{0!(3-0)!}.7^0.3^{3-0} = 0.027.$

$P(X=1) = \binom{3}{1}.7^1.3^{3-1} = \frac{3!}{1!(3-1)!}.7^1.3^{3-1} = 0.189.$

$P(X=2) = \binom{3}{2}.7^2.3^{3-2} = \frac{3!}{2!(3-2)!}.7^2.3^{3-2} = 0.441.$

$P(X=3) = \binom{3}{3}.7^3.3^{3-3} = \frac{3!}{3!(3-3)!}.7^3.3^{3-3} = 0.343.$

4.

x	P(X=x)	P(X≤x)
0	0.027	0.027
1	0.189	0.216
2	0.441	0.657
3	0.343	1.0

5. For you to do: Sketch the graph of the probability mass function for this Binomial random variable for the values 0 through 3 in the space provided below.

6. $EX = \mu = \sum xp(x) = np = 3(0.70) = 2.1$

7. $P(X \geq 2) = P(X = 2) + P(X = 3) = 0.441 + 0.343 = 0.784.$

8. There is almost an 80% chance of winning more than one of the games out of three games if the chance of winning any one game is 70%.

Exercise

Binomial Random Variable

Consider a parts manufacturing plant that builds very delicate computer parts. The chance of any one part being defective is 0.22. Five parts are randomly selected for testing every hour off of the manufacturing line.

1. What is the probability distribution of the number of defect parts out of the five chosen every hour if the defect rate is really 22%? State the name of the distribution and the parameter values of the distribution.
2. What is the variable X in this problem? Hint, X is the number of defective parts out of ....
3. What is the set of possible values for the random variable described in this problem?
4. Calculate the probability on each of the values of the random variable.
5. Form a probability distribution table with three columns, one for the values of the variable, one for the probabilities on each of the values and another column to list the cumulative probabilities.
6. Draw the graph which represents the probability distribution for this random variable.
7. What is the expected number of defective parts out of the five parts chosen each hour? Show equation & work.
8. What is the value of the standard deviation for the variable, number of defective parts out of five parts? Show the equation and the calculations required to obtain such a number.
9. What is the chance of no defective parts out of the five chosen in one hour with the 22% defect rate?
10. What is the chance of fewer than 2 defective parts out of the five chosen in one hour with the 22% defect rate?
11. What is the chance of more than 3 defective parts out of the five chosen in one hour with the 22% defect rate?

Exercise

Binomial Random Variable

The Oklahoma State University Basketball team has a 75% chance of beating its in-state rival university in each game the two teams play. Assume the two teams play four games during the season. The variable of interest is X=number of wins for OSU in the four games with the rival university.

1. What is the name of the probability distribution that describes the probability of such a variable?
2. State the values of the variable.
3. Calculate the probability of each value for the variable.
4. Present the probability distribution in a table form. Include the cumulative probability in an extra column. Notice the Binomial Table would have these cumulative entries if the text provided that combination of parameter values for n and p.
5. Present the probability distribution in a graph form.
6. What is the expected number of wins for OSU from these four games?
7. What is the chance of OSU winning at least two of the games out of these four games?
8. Write a sentence to be published in the O'Collegian concerning the probability calculated in part G.