Lesson 7
Discrete Random Variables

Learning Objectives

Upon completion of this lesson you should be able to do the following:

1. Express a random outcome as the value of a random variable and recognize whether the random variable is discrete or continuous.

2. Recognize that probability is represented by height in the graphs of distributions of discrete random variables and that probability is represented by area under the density curves for distributions of continuous random variables.

3. Recognize a discrete random variable and understand the idea of a discrete probability distribution.

4. Calculate the expected value and the variance of a discrete random variable based on the probability distribution for the variable.

Key Words

random variable, probability distribution, discrete, probability mass function, continuous, probability density function, expected value, mean, variance, uniform discrete

Concepts

The concept of a random variable was first introduced in the last lesson through the ideas associated with random phenomena. A **random variable** is a measured quantity associated with the random phenomenon. A random variable is a rule that assigns a number to each point in the sample space of an experiment. For example, toss three coins. Consider the number of heads out of three tosses. The number of heads in three tosses is a random variable that assigns the values of 0, 1, 2, or 3 to the possible outcomes of tossing three coins. If all three coins turned up heads, rather than tails, the value of the random variable would be three. What is the probability of observing 3 heads? That is the probability that the random variable is equal to 3. The values of random variables and the probability associated with the values are major concerns in statistics.

Probability Distributions

The **probability distribution** for a random variable tells the possible values of the random variable and the probability associated with the values or intervals of values. A **discrete** random variable has probability only on specific values. The probability distribution for a discrete random variable is called the **probability mass function,** because it tells how much of the mass of the unit of probability is at each specific value. A **continuous** random variable has probability on intervals of values, not on specific values. The probability distribution for a continuous random variable is called a **probability density function**, because it is the curve that tells how dense the probability is for a certain interval of values. Even though the details of working with probability distributions can be tricky the basic idea is simple. A probability distribution tells how the unit of probability is distributed or divided up among the possible values of the variable.

General Form of a Discrete Random Variable

A discrete probability distribution, called a probability mass function, tells the possible values of the variable and the probability associated with each value. This information can be given in the form of a table, function or graph. A table would list the values of the variable and the probability for each. The information can be stated in the form of a probability function where the domain of the function is the values that are possible for the variable. The value of the function at a certain value for the variable would be the probability for that value of the variable.

The graphs that represent discrete probability distributions use the height of a line to represent the probability at a specific value. The sum of the heights of all the lines on any certain distribution is equal to one, since the height of all of the lines together represent the unit of probability.

Some Notation

Capital X represents the random variable. Begin to think of the random variable as a description of what you are interested in or want to measure. **Lowercase** x represents the possible values of the variable. The probability on a certain value, x, of the random variable, X, is written as $P(X = x)$ or as $P(x)$. Remember x is a word or number; X is the description of the variable.

Expected Value and Variance of a Discrete Random Variable

The **expected value** of a random variable is not the most likely value of the variable but rather the **mean** of the values of the variable, or what we would expect over the long run. The expected value of a discrete variable does not even have to be one of the possible values of the variable. The expected value of a discrete variable X, designated as EX, is a weighted average of the possible values, each weighted by its associated probability. The **variance** of a random variable is the expected squared difference between the values of the variable and the mean of the variable. The concepts that you learned earlier about variance still hold here. The variance is a measure of dispersion about the mean in terms of squared units. The equations to calculate the expected value and the variance for a discrete random variable are shown below. The expected value of a random variable is indicated with the symbol μ to represent the mean of the probability distribution. The variance of a random variable is the symbol σ^2.

$\mu = EX = \sum xP(x)$, over all x.

$\sigma^2 = E(X-\mu)^2 = \sum(x-\mu)^2P(x) = \sum x^2P(x) - \mu^2$, over all x.

Uniform Discrete Random Variable

Assume that X is a discrete uniform random variable with possible values 1, 2,..., n. This means that the possible values for X are 1, 2 and so forth on the integers up to n. The probability distribution for a variable that is **uniform discrete** implies that each of the values has the same probability. The unit of probability is divided evenly among the possible values of the variable.

Consider the roll of one fair die. The number on the upward facing die side, after the roll, is a uniform discrete random variable. Assuming the die is a 'fair' die ensures that the values on the sides of the die, 1, 2, ..., 6 are equally likely. The random variable, X=number on the die roll, is a uniform discrete random variable since each of the six values receive an equal amount of the probability. Graphically, the probability mass function would be six lines, each of height 1/6 at the values 1, 2, ..., 6.

Uniform Discrete Random Variable: $X \sim Unf\,Dis(1,n)$.

The Variable: $X = a\ uniform\ discrete\ random\ variable\ with\ values\ on\ 1\ to\ n.$

Possible Values of the Variable: $x = 1,2,\ldots,n.$

Probability Mass Function: $f(x) = \frac{1}{n},\ x = 1,2,\ldots,n.$

Expected value: $\mu = EX = \sum xP(x) = \frac{(n+1)}{2}.$

Variance: $\sigma^2 = E(X-\mu)^2 = \sum(x-\mu)^2P(x) = \sum x^2P(x) - \mu^2 = \frac{(n^2-1)}{12}.$

Notice that the equations for the expected value and the variance are the same as they were for discrete distributions in general, but for a uniform discrete variable the equations for the mean and the variance reduce to $\frac{n+1}{2}$ and $\frac{n^2-1}{12}$, respectively.

Exercise with Answers

Discrete Random Variable

The weather forecast for rain precipitation for a local TV station on a specific day is described in the probability distribution given in the table below. The variable,

X = amount of rain in 24 hours, is measured to the closest tenth of inch increments.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6
probability	0.70	0.20	0.05	0.02	0.01	0.01	0.01

1. What is the probability of at least a tenth of an inch of rain on this day?
2. What is the expected amount of rain on this day?
3. What is the standard deviation for rain amounts on this day?
4. Draw a probability distribution graph for the variable X.
5. Label the mean of the distribution on the graph.
6. Write a sentence to describe the stated rain forecast to be read on TV for this day.
7. Write a sentence using the value 0.051.

Answers

1. $P(X \geq 0.10) = 1 - P(X = 0.0) = 1 - 0.70 = 0.30.$

2. $EX = \sum xP(X = x) = 0(0.70) + \ldots + .6(0.01) = 0.051$

3. $\sigma^2 = \sum x^2 P(X = x) - \mu^2 = 0^2(0.70) + \ldots + .6^2(0.01) - .051^2 =$
$= 0.0135 - 0.002601 = 0.010899, \sigma = 0.1044.$

4. For you to do: Sketch the graph of the probability mass function for this discrete random variable in the space provided below.

5. Label the value $\mu = 0.051$ on the graph you sketched above.

6. Today's forecast includes possible light showers with only a 30% of any measurable precipitation.

7. On average the expected amount of rainfall within this 24 hour period is 0.051 or only about 5 one-hundredths of an inch of rainfall.

Exercise

Discrete Random Variable

The weather forecast for rain precipitation for a local TV station on a specific day is described in the probability distribution given in the table below. The variable, X=amount of rain in 24 hours, is measured to the closest tenth of inch increments.

x	0.0	0.1	0.2	0.3	0.4	0.5	0.6
Probability	0.01	0.02	0.02	0.10	0.30	0.40	0.15

1. What is the probability of at least a tenth of an inch of rain on this day?
2. What is the expected amount of rain on this day?
3. What is the standard deviation for rain amounts on this day?
4. Draw a probability distribution graph for the variable X.
5. Label the mean of the distribution on the graph.
6. Write a sentence to describe the stated rain forecast to be read on TV for this day.
7. Write a sentence using the value of the expected amount of rainfall during this 24 hour period.