Lesson 6
Probability

Learning Objectives

Upon completion of this lesson you should be able to do the following:

1. Recognize a random phenomenon and understand the idea of probability as long-term relative frequency.

2. Identify a sample space for a random phenomenon.

3. Use in simple problems the basic laws of probability.

4. Calculate the probability of an event by assignment of probabilities to individual outcomes, by use of the addition rule, by use of the complement rule, or by use of the multiplication rule.

5. Recognize when it is reasonable to assume that two events are independent and what that characteristic implies about the probability of the intersection of those two events.

Key Words

likelihood, probability, sample space, random variable, probability distribution, experiment, simple event, event, compound events, union, intersection, complement, conditional probability, additive rule, mutually exclusive, disjoint, multiplicative rule, independent, random sample

Concepts

In statistics the **likelihood** of the observed outcome of an experiment is very important. The **probability** of an event is the likelihood of the event. In order to consider this likelihood we have to think of the set of all other outcomes that could occur. This set of all possible outcomes is called the **sample space**. For each outcome of an experiment or point in the sample space, some quantity of interest is measured. This quantity is a **random variable** and has an observable value for each outcome. The **probability distribution** associated with this random variable tells the possible values of the random variable and their likelihood based on the sample space of the experiment. In this lesson you will learn about these random variables and how to use various probability rules and equations to calculate probabilities associated with sets of events. First, probability will be considered as a form of relative frequency over many observations. For example, you know the probability of getting heads when a fair coin is tossed is one-half. The reason this is known is because if a coin tossd forever, half of the outcomes would be tails and the other half would be heads. You will also learn various tools to calculate the probability of unions and intersections. Do not try to memorize all these rules. You will want to include them on your formula sheet for the examination. Do try to understand them. When is it appropriate to use them? What types of questions do they answer?

This lesson focuses on measuring the likelihood of observable responses if certain characteristics are true about the population from which the responses are drawn. Each of you considers probability concepts every day. What is the chance of a certain thing happening? To begin to address this question we will concentrate on the basic terminology of probability first, then consider rules and equations associated with specific conditions.

Probability Terminology

An **experiment** is an act or process with an observable outcome that cannot be predicted with certainty. The **sample space** of an experiment is the set of all possible outcomes of the experiment. Each outcome of the experiment is a point in the sample space of the experiment and is called a **simple event**. An **event** is a collection of one or more simple events. The probability of an event is the likelihood of that event in the sample space of the experiment.

Compound events are comprised of two or more events. They are constructed from either the union or intersection of two or more events. The **union** of two events, A and B, is the set of all points either in event A, event B, or both. The **intersection** of two events, A and B, is the set of all points which are contained in both events A and B.

The **complement** of an event A is the set of all points in the sample space that are not contained in event A. The **conditional probability** of an event A conditioned upon the occurrence of event B is the probability of event A in the set of points that are contained in event B. The **additive rule** is a way to calculate the probability of the union of any two events in any situation.

Two events are **mutually exclusive** or **disjoint** if the two events have no points from the sample space in common. If two events are mutually exclusive then the occurrence of one of the events precludes the occurrence of the other event. In other words, if one of the events occurs the other event cannot occur.

The **multiplicative rule** is a way to calculate the probability of the intersection of any two events based on a conditional probability and one of the marginal probabilities. Two events are **independent** if the occurrence of one of the events does not alter the probability of the other event occurring.

A **random sample** is a sample chosen in such a way so that each element in the population has equal chance of being chosen. Each sample of size n would have equal chance to be chosen from the population of N observations if the sample were randomly chosen. The number of samples of size n that can be randomly drawn from a population with N observations is $\binom{N}{n} = \frac{N!}{n!(N-n)!}$.

Probability Rules

- $P(S) = P\{\text{ all outcomes }\} = 1$, where S is the sample space.
- $P(A) = \sum P(E)$, for all points E contained in Event A
- If A' is the complement of event A then $P(A) + P(A') = 1$.
- The conditional probability of event A given event B is $P(A|B) = P(A \cap B)/P(B)$
- Additive Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  From the Above: $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
- If events A and B are mutually exclusive then $P(A \cap B) = 0$.
- Multiplicative Rule: $P(A \cap B) = P(A|B)\ P(B)$

  equivalently, $P(A \cap B) = P(B|A)\ P(A)$
- If events A and B are independent then $P(A \cap B) = P(A)\ P(B)$

  equivalently, $P(A|B) = P(A)$ and $P(B|A) = P(B)$

Exercise One

Probability

Two hundred students were questioned about whether they will support Bush or Gore in the Presidential election if that turns out to be the parties choices. It was also recorded whether the students in the study were registered to vote as Republicans, Democrats, or not registered at all.

	Republican	Democrat	Not Registered
Supports Bush	46	18	54
Supports Gore	2	38	42

1. If a student is randomly selected what is the probability that the student is registered Democrat and plans to support Gore for President?
2. If a student who is not registered is chosen what is the probability that the student plans to support Bush?
3. What is the probability that a student who supports Bush is not registered to vote?
4. What is the probability that a student supports Bush and is not registered to vote?
5. What is the probability that a student is registered as a Republican or a Democrat?

Exercise Two

Probability

Four hundred shoppers in a large shopping mall were asked about the frequency of their shopping trips to the mall. The survey team also recorded the ages of the shoppers.

	Less than 21	Between 21 and 35	Between 35 and 70	More than 70
Trip once or more per week	62	28	12	22
Trip once per month	42	56	38	34
Trip once per year	31	39	16	20

1. What is the probability that a shopper is less than 21 years old?
2. What is the probability that if a shopper is between 35 and 70 years old that the shopper visits the shopping mall once per month?
3. What is the probability that a shopper more than 70 years old visits the shopping mall once or more per week?
4. Given that a shopper between the ages of 21 and 35 is chosen what is the probability that the shopper only visits a shopping mall once per year?
5. What is the probability that a shopper is between the ages of 21 and 35 and visits the shopping mall once or more per week?
6. What is the probability that a shopper visits the shopping mall once per month or is more than 70 years old?