Lesson 10

Continuous Random Variables

Learning Objectives

Upon completion of this lesson you should be able to do the following:

1. Recognize a continuous random variable and understand the idea of a continuous probability distribution.

2. Consider the expected value and the variance of a continuous random variable based on the graph of the probability distribution for the variable.

3. Recognize the uniform continuous distribution and calculate probabilities associated with intervals of values for the uniform continuous random variable.

Key Words

probability density function, continuous probability distribution, continuous random variables, uniform continuous random variable, uniform continuous distribution, uniform continuous probability density function

Concepts

The last three lessons dealt with discrete random variables that only have probability on specific values. This lesson introduces continuous random variables that have probability on intervals of values and no probability on single points. Recall that graphs for discrete probability mass functions are line graphs where the heights of the lines represents the probability at the single values of the variable. The graphs to represent probability for continuous random variables are curves. The probability for an interval of values is represented by the area under the curve. The area of a single line, which would represent the probability of a single value is zero. Thus, there is no probability on single values for a continuous random variable.

The probability distribution for a continuous random variable is called a **probability density function,** since they are curves that represent how dense the probability is for a certain interval of values. A **continuous probability distribution** spreads the unit of probability over one or more intervals of values. **Continuous random variables** differ from discrete random variables because they have positive probability only on intervals of values, not on specific values. The area under the density function for a specific interval is the probability that the random variable X has values in that interval. The total area between a continuous probability density function and the line known as the x-axis must be equal to one since this area represents the unit of probability.

The remainder of this lesson covers a specific kind of continuous random variable whose probability is spread evenly or uniformly across the interval of values possible. The probability density function is a flat line for the uniform continuous variable.

Uniform Continuous Random Variable

A **uniform continuous random variable** has the same probability for intervals of the same length inside the range of possible values for the variable. The **uniform continuous probability density function**, which is the probability distribution for this variable, is represented with a rectangle. The continuous uniform probability distribution is analogous to the discrete form of the uniform distribution. Recall that the discrete uniform probability distribution has equal probability for each of the possible values. Likewise, the continuous uniform probability distribution has equal probability on intervals of possible values of the same length.

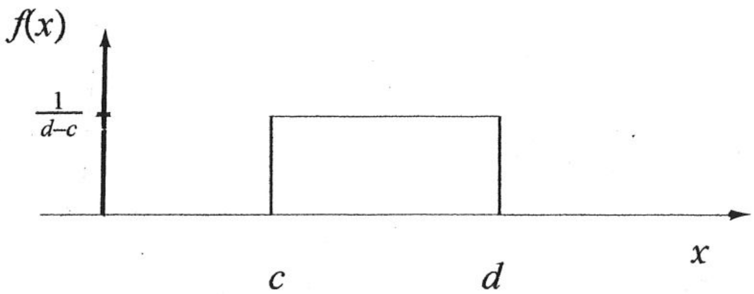
Assume that the variable X is a continuous uniform random variable distributed between the values, c and d. The probability for such a variable would be distributed evenly between c and d. For any interval of the same length between the values, c and d, the random variable X will have equal probability. The parameters for this distribution are the values of c and d. Recognize that c and d are the end points of the rectangle that represents the probability. The expected value and the variance of the uniform continuous variable are functions of the parameters c and d. Notice that the expected value, or mean, is just the average of the parameters c and d. The expected value for a continuous distribution is also the balance point of the probability distribution, just like it is for a discrete distribution. The expected value of the continuous uniform distribution is the center point between c and d where the rectangle would balance on your finger.

Uniform Continuous Random Variable with parameters c and d: $X \sim Unf(c, d)$

The Variable: $X =$ description of uniform continuous random variable.

Possible Values of the Variable: $c \leq x \leq d$

Uniform Probability Density Function: $f(x) = \frac{1}{d-c}, \quad c \leq x \leq d$



Expected value of X : $\mu = EX = \frac{c+d}{2}$,

Variance of X : $\sigma^2 = E(X-\mu)^2 = \frac{(d-c)^2}{12}$.

Exercise with Answers

Uniform Continuous Random Variable

A quality control statistician has determined that a flaw is equally likely to occur anywhere along the entire length of 8-foot steel beams during manufacturing. That is, the position of the flaw is uniformly distributed over the length of the 8-foot steel beams.

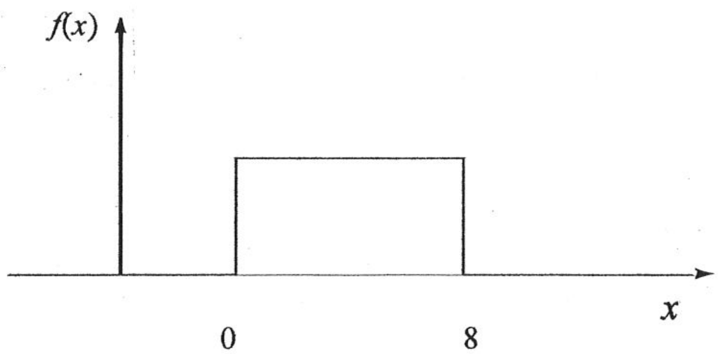
1. What is the name of the probability distribution that describes the probability of such a variable, flaw position? What is the interval of values for this variable?
2. What is the value of the probability density function for the variable, flaw position?
3. Present the probability density function in a graph.
4. What is the expected flaw position on these 8 foot steel beams? Label the value of the expected flaw position on the graph in number 3.
5. What is the chance that the flaw will be within one foot of either end of the 8-foot steel beams? Give a graph.
6. What is the probability that the flaw will be within 1.5 feet of the expected flaw position on the 8-foot steel beams? Give a graph.
7. If the flaws are bad then the manufacturer attempts to salvage 6-foot steel beams out of the 8-foot beams. What is the chance that a 6-foot beam can be saved?

Answers

1. The variable, $X = \mathit{flaw\ position}$, is uniformly distributed across the values 0' to 8'. That is, $X \sim Unf(0, 8)$. The possible values for this variable are all the values between 0 and 8, including 0 and 8. That is $0 \le x \le 8$.

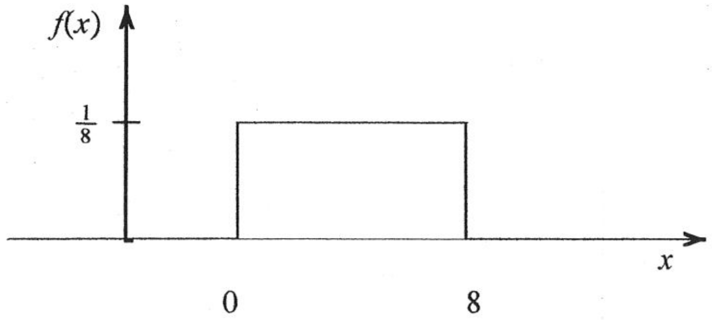
2. The value of the probability density function, the height of the rectangle that describes the probability for the variable, is the inverse of the length, $\frac{1}{8}$.

3. For you to do: label the $f(x)$ axis with the value of the height of the rectangle.

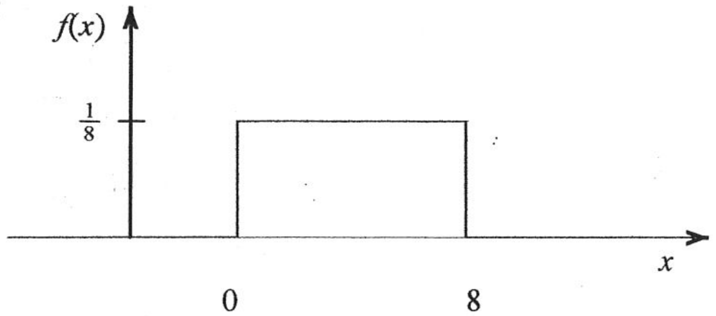


4. $EX = \mu = \frac{d+c}{2} = \frac{8+0}{2} = 4$. For you to do: Label the mean on the graph above.

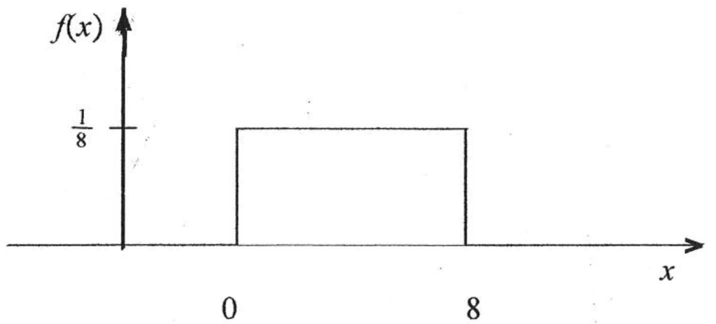
5. $P(X < 1) + P(X > 7) = \frac{1}{8}(1) + \frac{1}{8}(1) = \frac{2}{8}$. For you to do: Shade the area in the rectangle associated with the probability. Label the x-axis with the values of interest.



6. $P(\mu - 1.5 < X < \mu + 1.5) = P(2.5 < X < 5.5) = \frac{1}{8}(3) = \frac{3}{8}$. For you to do: Shade the area in the rectangle associated with the probability. Label the x-axis with the values of interest.



7. $P(X < 2) + P(X > 6) = \frac{1}{8}(2) + \frac{1}{8}(2) = \frac{1}{2}$. For you to do: Shade the area in the rectangle associated with the probability. Label the x-axis with the values of interest.



For you to do:

1. Label the height of the rectangle that indicates the probability density function for the variable in number 3..

2. Label the graph in number 3 with the mean from number 4.

3. Shade the area in the rectangle described by the probability statements in problems 5, 6, and 7. Label the x-axis with the values of interest.

Exercise

Uniform Continuous Random Variable

A quality control statistician has determined that a flaw is equally likely to occur anywhere along the entire length of 12-foot steel beams during the manufacturing. That is, the position of the flaw is uniformly distributed over the length of the 12 foot steel beams.

1. What is the name of the probability distribution that describes the probability of such a variable, flaw position? What is the interval of values for this variable?
2. What is the value of the probability density function for the variable, flaw position?
3. Present the probability density function in a graph.
4. What is the expected flaw position on these 12 foot steel beams? Label the value of the expected flaw position on the graph in number 3.
5. What is the chance that the flaw will be within two foot of either end of the 12-foot steel beams? Give a graph.
6. What is the probability that the flaw will be within 2.5 feet of the expected flaw position on the 12-foot steel beams? Give a graph.
7. If the flaws are bad then the manufacturer attempts to salvage 9-foot steel beams out of the 12-foot beams. What is the chance that a 9-foot beam can be saved?